1. (39) Suppose a matrix $A$ has an echelon form
\[
\begin{bmatrix}
1 & 0 & 3 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\] and $B$ has an echelon form
\[
\begin{bmatrix}
2 & 2 & 3 & 0 \\
0 & 3 & 2 & 0 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 4
\end{bmatrix}
\]. Answer the following questions. If there is not enough information to give an answer, say so.

a) How many pivots does $A$ have? _______

b) How many pivots does $B$ have? _______

c) How many solutions does $Ax = 0$ have? _______

d) How many solutions does $Bx = 0$ have? _______

e) How many solutions does $Ax = [1 \ 2 \ 3 \ 0]^T$ have? _______

f) How many solutions does $Bx = [1 \ 2 \ 3 \ 0]^T$ have? _______

g) Is the linear transformation $x \mapsto Ax$ one to one? _______

h) Is the linear transformation $x \mapsto Bx$ one to one? _______

i) Is the linear transformation $x \mapsto Ax$ onto? _______

j) Is the linear transformation $x \mapsto Bx$ onto? _______

k) Which of $A$ or $B$ is invertible? _______

l) One solution of $Ax = [1 \ 2 \ 3 \ 4]^T$ is $x = [0 \ 1 \ 0 \ 1]^T$. Find all solutions.

m) One solution of $Bx = [1 \ 2 \ 3 \ 4]^T$ is $x = [0 \ 1 \ 2 \ -1]^T$. Find all solutions.

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Signature

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2. (36)
   a) Suppose $A$, $B$, and $C$ are invertible $4 \times 4$ matrices. Solve for the $4 \times 4$ matrices $W$, $X$, $Y$ and $Z$.
   \[
   \begin{bmatrix}
   X & Y \\
   Z & W
   \end{bmatrix}
   \begin{bmatrix}
   0 & A \\
   B & C
   \end{bmatrix} = I_8
   \]
   
   b) Find $\begin{bmatrix} 0 & A \\ B & C \end{bmatrix}^{-1}$, with $A$, $B$, and $C$ invertible.

   c) Suppose that $T: \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation and also $T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$. Find $T(\begin{bmatrix} 2 \\ -1 \end{bmatrix})$. Also find the standard matrix of $T$. 
3. (25) Let \( \mathbf{v}_1 = [1 \ 2 \ 3 \ 1]^T \), \( \mathbf{v}_2 = [1 \ 0 \ -1 \ 1]^T \), and \( \mathbf{v}_3 = [1 \ 8 \ h \ 1]^T \). 

a) Find all \( h \) so that \( \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} \) is linearly dependent.

b) For each \( h \) you found in part a), determine if possible weights \( c_1 \) and \( c_2 \) so that \( \mathbf{v}_3 = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 \).

c) For each \( h \) you found in part a), determine whether or not \( \mathbf{v}_3 \) is in \( \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\} \).