# Pullback of the splitting to $\mathbb{E}^*$

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We use notation of [1]. Let $\widetilde{\mathbb{E}^*}$ be the pullback of the cover of $\mathbb{E}^1$ to $\mathbb{E}^*$ via the map $z \to z/\overline{z}$. The cocycle for $\widetilde{\mathbb{E}^*}$ is

$$c'(z,w) = c(z/\overline{z}, w/\overline{w})$$

Then any splitting $\zeta$ of the cocycle on $\mathbb{E}^1$ pulls back to a splitting $\zeta'$ of $c'$:

$$\zeta'(z) = \zeta(z/\overline{z})$$

In fact given $\zeta$, every splitting of $c'$ is of the form

$$\zeta'(z) = \zeta(z/\overline{z})\phi(z)$$

for a character $\phi$ of $\mathbb{E}^*$.

Recall $\zeta$ has a $\mu_n$ splitting where $n = 2, 4$ or $8$, depending on $\mathbb{F}$ and $\mathbb{E}$.

**Lemma 0.1** *The cocycle $c'(z,w)$ on $\mathbb{E}^*$ has a $\mu_2$ splitting.*

**Proof.** Let $\zeta$ be any splitting of the cocycle on $\mathbb{E}^1$. Then $\alpha(z) := \zeta(z)^2$ is a character of $\mathbb{E}^1$. This is an immediate consequence of the definition of a splitting, and the fact that $c(z,w) \in \mu_2$. Choose a character $\phi$ of $\mathbb{E}^*$ satisying:

$$\phi(z)^2 = \alpha(z/\overline{z})^{-1} \quad (z \in \mathbb{E}^*).$$

This is possible, since $\alpha((-z)/\overline{(-z)}) = \alpha(z/\overline{z})$. (That is, define $\phi(z^2) = \alpha(z/\overline{z})^{-1}$. This defines $\phi$ on $\mathbb{E}^{*2}$. Extend arbitrarily to $\mathbb{E}^*$.) Let

$$\zeta'(z) = \zeta(z/\overline{z})\phi(z)$$

Then

$$\begin{aligned}\zeta'(z)^2 &= \zeta(z/\overline{z})^2\phi(z^2)\\ &= \alpha(z/\overline{z})\phi(z^2)\\ &= 1\end{aligned}$$

■

# References

[1] Jeffrey Adams. Extensions of tori in sl(2). preprint.