MATH 241, 241H Fall 2008

Uniform Final Examination

Monday, 12/15/08

Instructions: Number the answer sheets from 1 to 8. Fill out all the information on the top of each sheet. Answer problem n on page n, $n = 1, \ldots, 8$. Do not answer one problem on more than one sheet. If you need more space use the back of the correct sheet. Please write out and sign the **Honor Pledge** on page 1 only.

SHOW ALL WORK

You May *Not* Use Calculators, Notes, Or Any Other Form Of Assistance On This Exam

1. (a) (10 pts) Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$. Find the projection of $\mathbf{b}$ onto $\mathbf{a}$.

   (b)(10 pts) Find the symmetric equations for the line that passes through the point $(1, 0, 2)$ and is perpendicular to the plane $3x + y = 7$.

2. The trajectory of a particle is given by

$$\mathbf{r}(t) = (\sin 2t)\,\mathbf{i} + (\cos 2t)\,\mathbf{j} + \frac{2}{3}t^{3/2}\,\mathbf{k} \quad \text{for} \quad t \geq 0.$$

   (a) (10 pts) Find the velocity and speed of this particle.

   (b) (10 pts) Find the total distance L traveled by the particle for $0 \leq t \leq 5$.

3. Consider the curve C parametrized by

$$\mathbf{r}(t) = (t^2 + 2)\mathbf{i} + (3t - 3)\mathbf{j} + (t^2 - 6t)\mathbf{k}.$$

   (a) (5 pts) Show that the point $P = (3, -6, 7)$ lies on C.

   (b) (10 pts) Find an equation of the line through P in the direction of the tangent to C at P.

4. (10 pts each) Consider the function $f(x, y, z) = xye^z$.

   (a) Find the gradient of f.

   (b) Compute the directional derivative $D_{\mathbf{u}}f$ at the point $(1, 1, 0)$, where $\mathbf{u}$ is the unit vector in the direction $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

   (c) Find a point on the level surface $f(x, y, z) = 1$ where the tangent plane is parallel to the plane $x + y + 2z = 3$.

EXAM CONTINUES ON THE OTHER SIDE

5. Consider the function $f(x, y) = x^2y - 2xy + 2y^2 - 15y$.

(a) (15 pts) Find all the critical points of f.

(b) (10 pts) Use the second partials test to classify the critical points found in (5a) above as relative maxima, relative minima, or saddle points.

6. (25 pts) Compute the double integral

$$I = \iint_R y \, dA ,$$

where R is the region of the first quadrant that is bounded above by the circle $x^2 + y^2 = 1$ and below by the parabola $y = 1 - x^2$.

7. (15 pts each) Consider the vector field

$$\mathbf{F} = 2xyz\,\mathbf{i} + zx^2\,\mathbf{j} + (x^2y + 1)\,\mathbf{k} .$$

(a) Is it possible to write $\mathbf{F} = \text{grad} f$ for some function f ? EXPLAIN. If the answer is yes, find this function f.

(b) Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r} ,$$

where C is the curve parametrized by

$$\mathbf{r}(t) = \cos(\pi t^3)\,\mathbf{i} + t^{5/3}\,\mathbf{j} + \frac{2t}{1+t^2}\,\mathbf{k} , \qquad 0 \le t \le 1 .$$

8. Consider the solid region D that lies above the xy-plane and is bounded above by the sphere $x^2+y^2+z^2 = 4$, below by the sphere $x^2+y^2+z^2 = 1$ and on the sides by the cone $3z^2 = x^2+y^2$.

(a) (20 pts) Compute the triple integral

$$\iiint_D (x^2 + y^2 + z^2)\, dV .$$

(b) (15 pts) Compute the flux integral

$$\iint_\Sigma \mathbf{F} \cdot \mathbf{n} \, dS ,$$

where $\mathbf{F}$ is the vector field $\mathbf{F} = x^3\,\mathbf{i} + y^3\,\mathbf{j} + z^3\,\mathbf{k}$, Σ is the boundary of region D and $\mathbf{n}$ is the unit outward normal vector on Σ. **Hint:** You might find part (8a) helpful.

END OF EXAM - GOOD LUCK!