MATH 241 Final Examination

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Monday, December 13, 2010

Instructions. Answer each question **on a *separate* answer sheet**. Show all your work. A correct answer without work to justify it may not receive full credit. *Be sure your name, section number, and problem number are on each answer sheet, and that you have copied and signed the honor pledge on the first answer sheet.* The point value of each problem is indicated. The exam is worth a total of 200 points. In problems with multiple parts, whether the parts are related or not, the parts are graded independently of one another. Be sure to go on to subsequent parts even if there is some part you cannot do. Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*
You are allowed use of one sheet of notes. Calculators are not permitted.

1. (20 points) Find the equation of the plane containing the points $(3, -1, -2)$, $(1, 1, 0)$, and $(0, 1, 2)$.

2. (30 points, 10 per part) This problem deals with the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ in the x-y plane, viewed as a curve C.

   (a) Give an explicit parameterization $\mathbf{r}(t)$ of C. (Trig functions are helpful.)

   (b) Find the formula for the curvature κ of C at the point (x, y). Recall that the curvature can be computed as $\kappa = \|\mathbf{v} \times \mathbf{a}\| / \|\mathbf{v}\|^3$.

   (c) Find the maximum and minimum values of κ, and find the points (x, y) where the maximum and minimum are obtained. Explain why your answers make intuitive sense in terms of the shape of the graph.

3. (25 points) Evaluate the iterated integral: $\displaystyle \int_0^1 \int_y^1 e^{x^2}\, dx\, dy$.

4. (25 points) Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 16$ that is inside the cylinder $x^2 + y^2 = 1$.

5. (25 points) Evaluate the surface integral $\iint_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS$, where
$$\mathbf{F}(x, y, z) = xy\,\mathbf{i} - y\,\mathbf{j} + (1 + z)\,\mathbf{k}$$
and Σ is the boundary of the solid region in the first octant bounded by the coordinate planes, the plane $z = 1 + x$, and the parabolic sheet $x = 1 - y^2$, and $\mathbf{n}$ is the outward pointing normal.

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6. (25 points, 10 for (a) and 15 for (b))

   (a) Show that the surfaces described by the equations $z = 4x^2 + y^2 - 2$ and $z = x^2 + 4y - 6$ have the same tangent plane at the point $(0, 2, 2)$.

   (b) Find the points on the surface $x^3 + y - z^2 = 10$ at which the plane tangent to this surface is parallel to the plane $27x + y - 8z = 4$.

7. (25 points) Consider the function

$$f(x, y) = y\sqrt{1 + x} + x\sqrt{1 + y} \qquad \text{where } x > -1 \text{ and } y > -1.$$

Find all critical points (x, y) of $f(x, y)$ with $x > -1$ and $y > -1$, and characterize each one as a relative maximum, relative minimum, or a saddle point.

8. (25 points, 10 for (a) and 15 for (b))

   (a) Show that the vector field $\mathbf{F} = (x^2 + y^2)\,\mathbf{i} + 2xy\,\mathbf{j} + 3z\,\mathbf{k}$ is conservative.

   (b) Compute the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the portion of the curve $z = 4 - x^2$, $y = x$ from $(-2, -2, 0)$ to $(2, 2, 0)$.