Instructions: Number the answer sheets from 1 to 9. Fill out all the information at the top of each sheet. Answer problem n on page n, $n = 1, \cdots, 9$. Do not answer one question on more than one sheet. If you need more space use the back of the correct sheet. Please write out and sign the **Honor Pledge** on page 1 only.

SHOW ALL WORK

The Use of Calculators Is Not Permitted On This Exam

1. (20 points) Let $A = (0, 0, 0)$, $B = (1, 0, 0)$, $D = (1, 2, 2)$, $E = (0, 2, 2)$.
(a) Show that these four points lie on a plane and find an equation of that plane.
(b) Sketch the quadralateral C whose vertices are A, B, D and E.
(c) Show that C is a parallelogram.
(d) Show that C is a rectangle.
(e) Is C a square? Explain.

2. (25 points) Let $A = (3, 2, 0)$, $B = (6, 1, 2)$.
(a) Find parametric equations for the line L containing A and B.
(b) Let $\mathbf{F} = 2y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$. Find the work W done by the force $\mathbf{F}$ on an object moving from A to B along L.

3. (25 points) The position of a moving particle is given by

$$\mathbf{r}(t) = t^2\,\mathbf{i} + 2t\,\mathbf{j} + \ln t\,\mathbf{k} \text{ for } 2 \leq t \leq 4.$$

(a) Find the velocity, speed, and the tangential and normal components of the acceleration of the particle for any t with $2 \leq t \leq 4$.
(b) Find the total distance travelled by the particle in the given time interval.

4. (20 points) Let

$$f(x, y, z) = 2x^3 + y - z^2$$

(a) Find the points on the level surface $f(x, y, z) = 5$ at which the tangent plane is parallel to the plane $24x + y - 6z = 3$.
(b) Find the directional derivative of f at the point $P = (1, 1, 2)$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.
(c) In what direction is the directional derivative of f a maximum at P and what is the value of the maximum?

5. (20 points) Suppose that a firm makes two products, widgets and flibbits, using the same raw materials. If x widgets and y flibbits are produced then x and y must satisfy the constraint $x^2 + 2y^2 = 8100$. (This expresses a limitation on the amount of raw materials available.) Each widget produces \$5 profit and each flibbit produces \$20 profit. How many of each product should the firm produce in order to maximize the profit ?

6. (20 points) Write a triple integral in an appropriate coordinate system for the volume V of the solid region bounded above by the sphere $x^2 + y^2 + z^2 = 49$ and below by the paraboloid $x^2 + y^2 = 3z + 21$. Do not evaluate the integral but, if you used the right coordinate system, you should observe that the integration is not particularly difficult.

7. (20 points) Evaluate

$$\int\int_D \frac{x + 2y}{(x - 2y)^2}\, dA$$

where D is the region bounded by the lines $x + 2y = 1$, $x + 2y = 3$, $x - 2y = 4$ and $x - 2y = 8$ by making an appropriate change of variables.

8. (25 points) Compute $\int\int_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS$ where

$$\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$$

and Σ is the boundary of the part of the ball $x^2 + y^2 + z^2 \leq 100$ which lies in the first octant ($x > 0$, $y > 0$, $z > 0$) and $\mathbf{n}$ is the outward normal.

9. (25 points) Use Stokes's theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = z^2\mathbf{i} + 4x\mathbf{j} + y^3\mathbf{k}$$

and C is the rectangle of Problem 1 oriented counterclockwise as viewed from above.