Math 241 **Final Exam** **Spring 2010**

This exam has **8 questions.**

<u>**Instructions**</u>: Number the answer sheets from 1 to 8. Fill out **all** the informations at the top of **each** sheet (write and sign the Honor Pledge on page 1 only). Answer **one** question on **each** sheet in the correct order. (Do **not** answer one question on more than one sheet. Use the back of the correct sheet if you need more space).

None of the following are allowed: lecture notes, book, electronic devices of any kind (including calculators, cell phones, etc.)

You may keep with you **one sheet of handwritten notes.**

1. [25 points] Consider the line ℓ_0 with symmetric equations

$$\frac{x+2}{2} = \frac{y-4}{3}, \qquad z = -1$$

and the line ℓ_1 with symmetric equations

$$\frac{x-1}{3} = \frac{y-2}{-2} = \frac{z-2}{3}$$

   (a) Show that ℓ_0 and ℓ_1 are perpendicular.

   (b) Find the point of intersection P_0 of the lines ℓ_0 and ℓ_1.

   (c) Find an equation of the plane containing both ℓ_0 and ℓ_1.

2. [25 points] Consider the curve $\mathcal{C}$ parametrized by

$$\vec{r}(t) := 2t\,\vec{\imath} + t^2\,\vec{\jmath} + \ln t\,\vec{k}, \qquad 1 \le t \le 4.$$

   (a) Explain why $\mathcal{C}$ is a smooth curve

   (b) Find parametric equations for the line passing through $P_0 := \vec{r}(2)$ in the direction of the tangent to $\mathcal{C}$ at P_0.

   (c) Find the length of the curve $\mathcal{C}$.

3. [25 points] Consider the function

$$f(x,y) := x^2y - x^2 - 2y^2 + 3$$

   (a) Find all critical points of the function f

   (b) Determine whether each critical point yields a relative maximum value, a relative minimum value or a saddle point.

4. [20 points] Let

$$f(x,y,z) := ze^{x^2-y^2}$$

   (a) Find the directional derivative of f at the point $(1,1,1)$ in the direction of

$$\vec{a} := 2\vec{i} - \vec{j} - 2\vec{k}.$$

   (b) Find the unit vector for which the directional derivative is maximal at the point $(1,1,1)$.

5. [30 points] Compute the double integral

$$\iint_R \frac{1}{1+x^4} dA$$

where R is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$. **Hint**: Integrate first in the y variable.

6. [25 points] Compute the triple integral

$$\iiint_D (x^2+y^2)dV$$

where D is the region bounded above by the paraboloid $z = 25 - x^2 - y^2$ and below by the x-y plane.

7. [25 points] Use Green's theorem to compute the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F} := y^3\vec{i} + x^3\vec{j}$ and C is the triangle with vertices $(0,0)$, $(1,0)$ $(0,1)$ oriented counter-clockwise.

8. [25 points] Use Gauss's theorem (also known as the divergence theorem) to compute the flux

$$\iint_S (\vec{F} \cdot \vec{n})dS$$

where $\vec{F} := x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and S is the boundary of the region D bounded above by the sphere $x^2 + y^2 + z^2 = 9$ and below by the x-y plane. $\vec{n}$ is the unit normal vector directed outward from the region D.