Math 241 Exam 1 Sample 3 Solutions

1. (a) We have $\bar{PQ} = 2\,\hat{\imath} - 2\,\hat{\jmath} - 1\,\hat{k}$ and to make it length 1 we:

$$\frac{\bar{PQ}}{||\bar{PQ}||} = \frac{2\,\hat{\imath} - 2\,\hat{\jmath} - 1\,\hat{k}}{\sqrt{4+4+1}}$$

(b) We need

$$\begin{aligned}(\alpha\,\hat{\imath} - 2\,\hat{\jmath} + \alpha\,\hat{k}) \cdot (2\,\hat{\imath} + 5\,\hat{\jmath}) &= 0\\ 2\alpha - 10 &= 0\\ \alpha &= 5\end{aligned}$$

(c) We have

$$\text{Pr}_{\bar{b}}\bar{a} = \frac{\bar{b}\cdot\bar{a}}{\bar{b}\cdot\bar{b}}\bar{b} = \frac{2+10}{1+4+9}(1\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k})$$

2. (a) The plane has $\bar{N} = 2\,\hat{\imath} + 3\,\hat{\jmath} - 1\,\hat{k}$ and a point is $P = (2, 0, 0)$ (any points satisfying the equation). Then with $Q = (3, 2, 1)$ we have $\bar{PQ} = 1\,\hat{\imath} + 2\,\hat{\jmath} + 1\,\hat{k}$ and so

$$d = \frac{|\bar{N} \cdot \bar{PQ}|}{||\bar{N}||} = \frac{|2 + 6 - 1|}{\sqrt{4 + 9 + 1}}$$

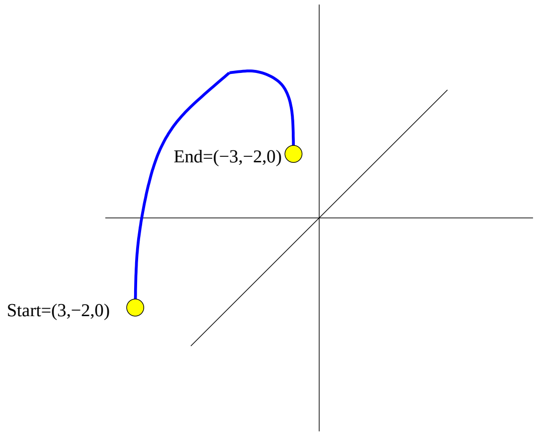
(b) First:

$$\begin{aligned} \bar{r}(t) &= t\,\hat{\imath} + \sin t\,\hat{\jmath} \\ \bar{v}(t) &= 1\,\hat{\imath} + \cos t\,\hat{\jmath} \\ \bar{a}(t) &= 0\,\hat{\imath} - \sin t\,\hat{\jmath} \end{aligned}$$

Then $||\bar{a} \times \bar{v}|| = \sin t\,\hat{k}$ and so

$$\begin{aligned} \kappa(t) &= \frac{||\bar{a} \times \bar{v}||}{||\bar{v}||^3} \\ \kappa(t) &= \frac{\sqrt{\sin^2 t}}{(1 + \cos^2 t)^{3/2}} \\ \kappa(\pi/2) &= \frac{1}{(1 + 0)^{3/2}} \end{aligned}$$

3. (a) The graph is:



(b) The parabolic part is $\bar{r}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath}$ for $-1 \leq t \leq 2$.

The straight part is $\bar{r}(t) = (2-3t)\,\hat{\imath} + (4-3t)\,\hat{\jmath}$ for $0 \leq t \leq 1$.

4. (a) The vector is $\bar{L} = 3\,\hat{\imath} - 4\,\hat{\jmath} - 2\,\hat{k}$ and so using the first point we have

$$\frac{x-1}{3} = \frac{y-2}{-4} = \frac{z-3}{-2}$$

(b) Start with:

$$\begin{aligned}
\bar{a}(t) &= 2\,\hat{\imath} \\
\bar{v}(t) &= \int \bar{a}(t)dt = 2t\,\hat{\imath} + \bar{C} \\
\bar{v}(1) &= 2\,\hat{\imath} + \bar{C} = 2\,\hat{\imath} + \,\hat{\jmath} + \,\hat{k} \\
\bar{C} &= \,\hat{\jmath} + \,\hat{k}
\end{aligned}$$

Therefore

$$\begin{aligned}
\bar{v}(t) &= 2t\,\hat{\imath} + \,\hat{\jmath} + \,\hat{k} \\
\bar{r}(t) &= \int \bar{v}(t)dt = t^2\,\hat{\imath} + t\,\hat{\jmath} + t\,\hat{k} + \bar{D} \\
\bar{r}(1) &= 1\,\hat{\imath} + 1\,\hat{\jmath} + 1\,\hat{k} + \bar{D} = \bar{0} \\
\bar{D} &= -1\,\hat{\imath} - 1\,\hat{\jmath} - 1\,\hat{k}
\end{aligned}$$

And so finally

$$\bar{r}(t) = (t^2 - 1)\,\hat{\imath} + (t-1)\,\hat{\jmath} + (t-1)\,\hat{k}$$

5. Let's find the line through $(1, 2, 3)$ which is perpendicular to the plane and see where it hits the plane. If it's perpendicular it has the vector $\bar{L} = \bar{N} = 2\,\hat{\imath} + 3\,\hat{\jmath} + 1\,\hat{k}$ and so the line is

$$\begin{aligned} x &= 1 + 2t \\ y &= 2 + 3t \\ z &= 3 + t \end{aligned}$$

Hitting the plane means satisfying the equation:

$$\begin{aligned} 2(1 + 2t) + 3(2 + 3t) + (3 + t) &= 8 \\ 2 + 4t + 6 + 9t + 3 + t &= 8 \\ 14t &= -3 \\ t &= -3/14 \end{aligned}$$

So this is at the point

$$\begin{aligned} x &= 1 + 2(-3/14) \\ y &= 2 + 3(-3/14) \\ z &= 3 + (-3/14) \end{aligned}$$