

1. Given the following data:

$$\begin{aligned}P &= (-1, 0, 3) \\Q &= (2, 5, 5) \\ \bar{a} &= 1 \hat{i} + 2 \hat{j} + 0 \hat{k} \\ \bar{b} &= 3 \hat{i} + 2 \hat{j} + 1 \hat{k}\end{aligned}$$

(a) We use:

$$2 \left(\frac{\vec{PQ}}{\|\vec{PQ}\|} \right) = 2 \left(\frac{3 \hat{i} + 5 \hat{j} + 2 \hat{k}}{\sqrt{9 + 25 + 4}} \right)$$

(b) We have:

$$\begin{aligned}\bar{a} \cdot \bar{b} &= \|\bar{a}\| \|\bar{b}\| \cos \theta \\ 7 &= \sqrt{1 + 4 + 0} \sqrt{9 + 4 + 1} \cos \theta \\ \cos \theta &= \frac{7}{\sqrt{5} \sqrt{14}}\end{aligned}$$

(c) We have:

$$\begin{aligned}\text{Pr}_{\bar{a}} \bar{b} &= \left(\frac{\bar{a} \cdot \bar{b}}{\bar{a} \cdot \bar{a}} \right) \bar{a} \\ &= \left(\frac{7}{5} \right) (1 \hat{i} + 2 \hat{j} + 0 \hat{k})\end{aligned}$$

2. (a) The line has $\vec{L} = 3\hat{i} + 0\hat{j} + 1\hat{k}$ and $P = (-2, -2, 3)$. We have $Q = (3, 2, 1)$ off the line. Therefore

$$\vec{PQ} = 5\hat{i} + 4\hat{j} - 2\hat{k}$$

and so the distance is:

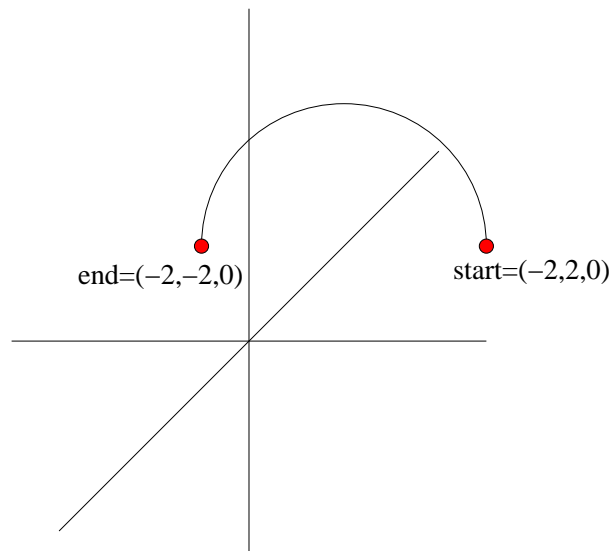
$$\begin{aligned}\text{Dist} &= \frac{\|\vec{L} \times \vec{PQ}\|}{\|\vec{L}\|} \\ &= \frac{\| -4\hat{i} + 11\hat{j} + 12\hat{k} \|}{\sqrt{9 + 1 + 0}} \\ &= \frac{\sqrt{16 + 121 + 144}}{\sqrt{9 + 1 + 0}}\end{aligned}$$

- (b) We have

$$\vec{r}'(t) = -\sin(t)\hat{i} + 3\cos(t)\hat{j}$$

$$\begin{aligned}\vec{T}(\pi/4) &= \frac{\vec{r}'(\pi/4)}{\|\vec{r}'(\pi/4)\|} \\ &= \frac{-\sin(\pi/4)\hat{i} + 3\cos(\pi/4)\hat{j}}{\|-\sin(\pi/4)\hat{i} + 3\cos(\pi/4)\hat{j}\|} \\ &= \frac{-\frac{\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}}{\|-\frac{\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}\|} \\ &= \frac{-\frac{\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}}{\sqrt{\frac{1}{2} + \frac{9}{2}}} \\ &= \frac{-\frac{\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}}{\sqrt{5}} \\ &= \frac{\sqrt{10}}{10}\hat{i} + \frac{3\sqrt{10}}{10}\hat{j}\end{aligned}$$

3. (a) We have



(b) We have three pieces:

$$\bar{r}(t) = t \hat{i} + (4 - t^2) \hat{j} \quad \text{with } 0 \leq t \leq 2$$

$$\bar{r}(t) = (2 - t) \hat{i} + 0 \hat{j} \quad \text{with } 0 \leq t \leq 2$$

$$\bar{r}(t) = 0 \hat{i} + t \hat{j} \quad \text{with } 0 \leq t \leq 4$$

4. (a) If it's perpendicular to both then we can use the cross product so

$$\bar{L} = (2\hat{i} + 2\hat{j} + 0\hat{k}) \times (3\hat{i} - 1\hat{j} + 2\hat{k}) = 4\hat{i} - 4\hat{j} - 8\hat{k}$$

and so we have

$$x = 1 + 4t$$

$$y = 2 - 4t$$

$$z = 3 - 8t$$

- (b) The line hits the plane when

$$2(2t + 2) + (5 - t) - (t + 10) = 1$$

$$4t + 4 + 5 - t - t - 10 = 1$$

$$2t = 2$$

$$t = 1$$

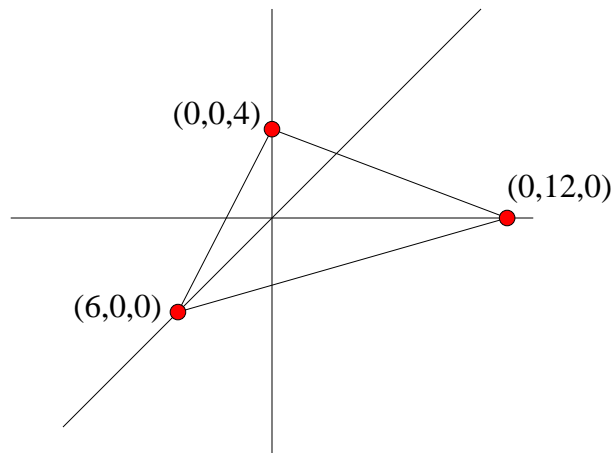
and this occurs at the point given by

$$\bar{r}(1) = 4\hat{i} + 4\hat{j} + 11\hat{k}$$

which is

$$(4, 4, 11)$$

5. (a) We have



(b) We have

$$\vec{r}'(t) = 2t \hat{i} + 3t^2 \hat{j} + 0 \hat{k}$$

and so the length is

$$\int_0^1 \|\vec{r}'(t)\| dt = \int_0^1 \sqrt{4t^2 + 9t^4 + 0} dt$$