Math 241 Sections 01 Exam 2** **Dr. Justin O. Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Use the gradient to find a vector perpendicular to the graph of the curve $y = x^3 + x - 2$ at the point where $x = 2$. [10 pts]

   (b) Suppose the base of a triangle is growing at 2 inches per hour while the height is growing at 3 inches per hour. At what rate is the area growing when the height is 10 inches and the base is 20 inches? [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Sketch the graph of the surface $y^2 = x^2 + z^2$. Write the name. [5 pts]

   (b) Sketch the graph of the surface $y = x^2$. Write the name. [5 pts]

   (c) Find the directional derivative of $f(x, y) = y \sin(xy)$ in the direction of $\bar{a} = 2\,\hat{\imath} + \,\hat{\jmath}$ at the point $\left(\frac{\pi}{8}, 2\right)$. Simplify. [10 pts]

Please put problem 3 on answer sheet 3

3. (a) All together on one graph sketch the level curves for $f(x, y) = y - |x|$ at $c = -2, 0, 2$ and label each with its value of c. [5 pts]

   (b) Suppose the unit vector $\bar{u}$ makes an angle of $30°$ with the gradient of a function f at $(1, 2)$ and $||\nabla f(1, 2)|| = 3$. Find $D_{\bar{u}} f(1, 2)$. [5 pts]

   (c) The function $f(x, y) = x^2 y - 2x^2 - y^2$ has the following: [10 pts]

   $$f_{xx}(x, y) = 2y - 4 \qquad f_{yy}(x, y) = -2 \qquad f_{xy}(x, y) = 2x$$

   There are three critical points at $(0, 0)$, $(2, 2)$ and $(-2, 2)$. Categorize each critical point as a relative maximum, relative minimum or saddle point.

Please put problem 4 on answer sheet 4

4. Find the maximum and minimum values of $f(x, y) = x^2 + 2y^2$ on the quarter circle $x^2 + y^2 \leq 4$ with $x, y \geq 0$. [20 pts]

Please put problem 5 on answer sheet 5

5. Let $f(x, y) = x^2 + 6y^2$ and suppose (x, y) is constrained by $x + 3y = 10$.

   (a) Use Lagrange multipliers to find the minimum of $f(x, y)$ subject to the constraint. [16 pts]

   (b) Explain why $f(x, y)$ has no maximum subject to the constraint. [4 pts]

The End