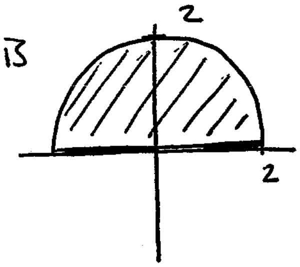
1(a) The region R is



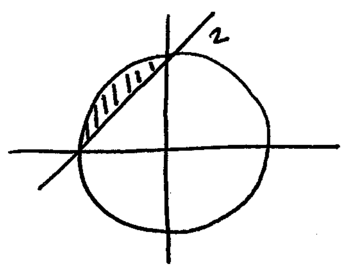
The semicircle breaks into a left and right fctn

$$x^2+y^2=4$$

$$x=\pm\sqrt{4-y^2}$$

So the integral is $\displaystyle\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x \, dx \, dy$

(b) The region R is



the circle $r=2$ is the outside function while the line is the inside function

$$y=x+2$$

$$y-x=2$$

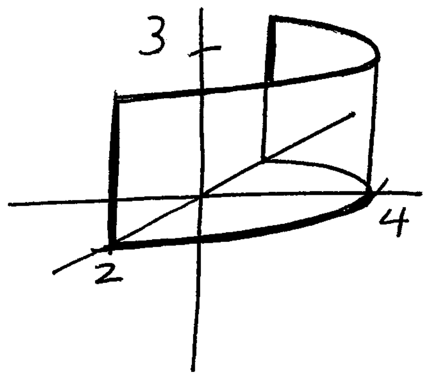
$$r\sin\theta - r\cos\theta = 2$$

$$r=\frac{2}{\sin\theta-\cos\theta}$$

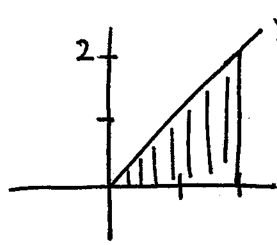
So the integral is

$$\int_{\frac{\pi}{2}}^{\pi} \int_{\frac{2}{\sin\theta+\cos\theta}}^{2} r\sin\theta \, r \, dr \, d\theta$$

2(a) Since $y = 4 - x^2$ and z is left alone the surface is a piece of the parabolic sheet



(b) Since we can't integrate directly wrt x we change the order of integration. The region R is



as vs this is

$$\int_0^2 \int_0^x e^{(x^2)}\,dy\,dx$$

$$= \int_0^2 x e^{(x^2)}\,dx$$

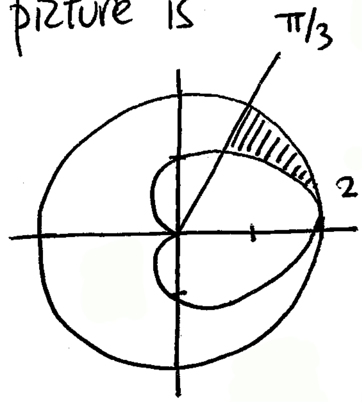
$$= \frac{1}{2} e^{(x^2)} \Big|_0^2$$

$$= \frac{1}{2}e^4 - \frac{1}{2}$$

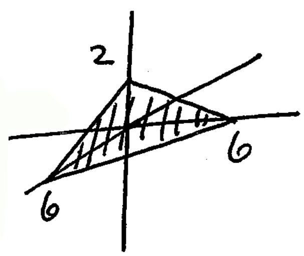
3(a) The outside fctn is $r=2$, a circle.

The inside fctn is $r=1+\cos\theta$, a cardioid.

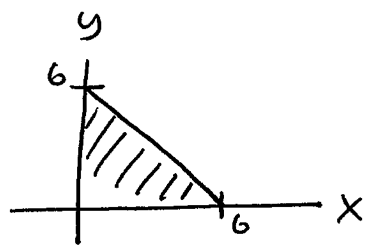
Thus the picture is



(b) The picture (not necessary) is



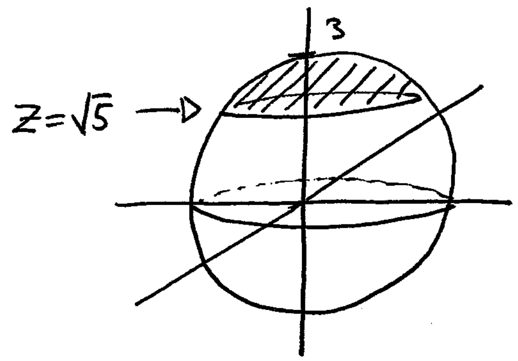
and R is

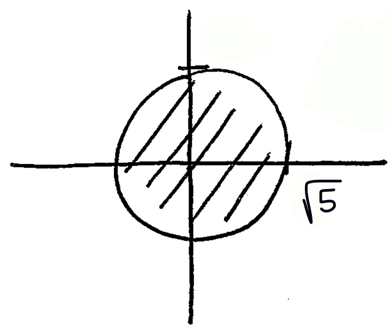


The volume is $\iiint_D 1\,dV$

$$= \int_0^6 \int_0^{6-x} \int_0^{\frac{1}{3}(6-x-y)} 1 \, dz\, dy\, dx$$

4(a) Here are D and R



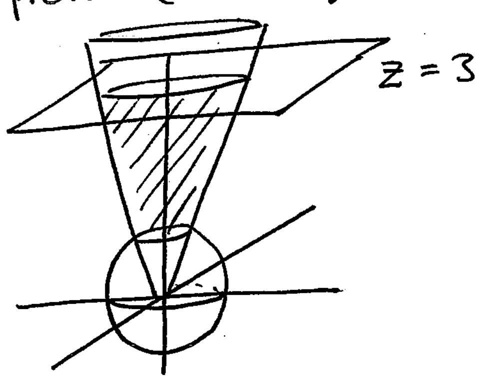


↑ meet at: $x^2+y^2+(\sqrt{5})^2=9$

$x^2+y^2=4$

So the integral is $\iiint_D 1\,dV = \int_0^{2\pi}\int_0^{2}\int_{\sqrt{5}}^{\sqrt{9-r^2}} 1\cdot r\,dz\,dr\,d\theta$

(b) The picture (not nec.) is



the inner fctn is the sphere $\rho=1$

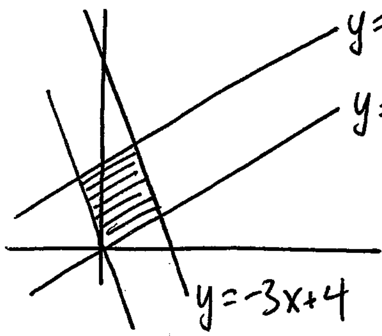
the outer fctn is the plane $z=3$ or $\rho\cos\phi=3$ or $\rho=3\sec\phi$

the cone is $\phi=\pi/6$

The mass is then

$$\iiint_D 1\,dV = \int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_1^{3\sec\phi} 1\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

5. The region R is



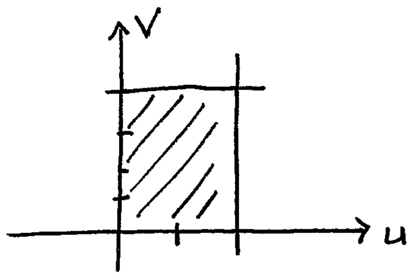
$y = x+2 \Rightarrow y - x = 2$

$y = x \Rightarrow y - x = 0$

$y = -3x+4 \Rightarrow 3x + y = 4$

$y = -3x \Rightarrow 3x + y = 0$

we put $u = y - x$ and $v = 3x + y$ so the lines become $u=2, u=0, v=4, v=0$ and S is



The Jacobian is $1 \div \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} = 1 \div (-4) = -\frac{1}{4}$

we need x for the integrand. If $y = u + x$

then $v = 3x + (u+x)$

$4x = v - u$

$x = \frac{1}{4}(v-u)$

$y = x + u = \frac{1}{4}(v-u) + u$

$= \frac{1}{4}v + \frac{3}{4}u$

And so

$$\iint_R x \, dA = \iint_S \frac{1}{4}(v-u) \cdot \left|-\frac{1}{4}\right| dA = \int_0^2 \int_0^4 \frac{1}{16}(v-u)\, dv\, du$$