Math 241 Exam 4 Sample 3

Directions: Do not simplify or evaluated unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. A certain filter Σ is in the shape of the parabolic sheet $z = 9 - x^2$ with $0 \le x \le 2$, $0 \le y \le 2$ and has upwards orientation. Find the rate at which the fluid $\bar{F}(x, y, z) = 0\,\hat{\imath} + x\,\hat{\jmath} + z\,\hat{k}$ is passing through Σ. [20 pts]
   Stop when you have an unsimplified numerical answer.

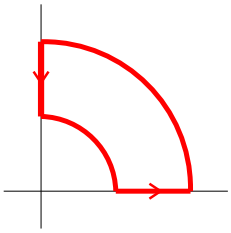
Please put problem 2 on answer sheet 2

2. (a) Evaluate $\int_C xy^2 + 1\ dx + x^2 y\ dy$ where C is an unknown curve from $(1, -2)$ to $(3, 3)$. [10 pts]
   Stop when you have a simplified numerical answer.

   (b) Find the line integral of the function $f(x, y) = 2x + y$ over the straight line segment C joining the points $(0, 0)$ to $(5, 4)$. [10 pts]
   Stop when you have an iterated single integral.

Please put problem 3 on answer sheet 3

3. Evaluate $\int_C 2x\ dx + x^2\ dy$ where C is the oriented curve shown in the picture. The non-straight parts are along the circles of radius 1 and 2. [20 pts]
   Stop when you have an unsimplified numerical answer.



Please put problem 4 on answer sheet 4

4. Let C be the intersection curve of the plane $x + y = 5$ with the cylinder $x^2 + z^2 = 4$ with counterclockwise orientation when viewed toward the origin along the y-axis. Pick an appropriate Σ and use Stokes's Theorem to evaluate $\int_C x\ dx + 3\ dy + 2y\ dz$. [25 pts]
   Stop when you have an iterated double integral.

Please put problem 5 on answer sheet 5

5. Let Σ be the surface of the outward-oriented cube with opposite corners $(0, 0, 0)$ and $(2, 2, 2)$. Find $\iint_\Sigma (5x\,\hat{\imath} + 2y\,\hat{\jmath} - 2z\,\hat{k}) \cdot \bar{n}\ dS$. [15 pts]
   Stop when you have a simplified numerical answer.

The End!