

Spring 2009: Math 241 (Section 02); Practice EXAM 1

1. Consider the following vectors:

$$\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}, \quad \vec{b} = 2\vec{i} + 2\vec{j} + 10\vec{k}, \quad \text{and } \vec{c} = 4\vec{i} + 5\vec{j} + 9\vec{k}.$$

- (a.) Show that \vec{a} and \vec{b} are perpendicular.
(b.) Resolve \vec{c} into vectors parallel to \vec{a} and \vec{b} .

2. Find the equation of the plane containing the point $P_0 = (-1, -1, -1)$ and the vectors

$$\vec{i} - \vec{j} + 2\vec{k}, \quad \text{and } -2\vec{i} + 3\vec{j} - \vec{k}.$$

3. Consider the plane \mathcal{P} defined by the equation

$$-x + 3y + 2z = -6.$$

- (a.) Prove that the point $P_0 = (1, -1, -1)$ lies on the plane \mathcal{P} .
(b.) Find parametric equations of the line ℓ perpendicular to the plane \mathcal{P} and going through the point P_0 .

4. (a.) An object leaves the point $(0, 0, 1)$ with initial velocity $\vec{v}_0 = 2\vec{i} + 3\vec{k}$. Thereafter it is subject only to the force of gravity. Find a formula for the position of the object at any time $t > 0$.

(b.) Find the length L of the curve C parametrized by the vector-valued function

$$\vec{r}(t) = \sin t\vec{i} - \sqrt{2}\cos t\vec{j} + \sin t\vec{k}, \quad 0 \leq t \leq \pi/2.$$