Spring 2009: Math 241 (Section 02); Practice EXAM 2

1. Let $\vec{r}(t) = t\vec{\imath} + \frac{2\sqrt{2}}{3}t^{3/2}\vec{\jmath} + \frac{1}{2}t^2\vec{k}$ be a parametrization of a curve.
(a.) Find the length of the portion of the curve that lies between $\vec{r}(0)$ and $\vec{r}(1)$.
(b.) Find the tangential and the normal components of the acceletation vector.
(c.) What is the radius of curvature at the point $\vec{r}(1)$?

2. Let $z = e^{-y}\cos x$. Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

3. Consider the function $f(x, y) = 4 - x^2 + 3y^2 + y$.
(a.) Find the direction in which f increases most rapidly at the point $(-1, 0)$. What is the maximum directional derivative of f at this point?
(b.) Let S be the surface described by the equation $z = f(x, y)$ where $f(x, y)$ is given above. Find an equation for the plane tangent to S at the point $(-1, 0, 3)$.

4. Let $z = x^2 + y^2$ and $x = r\cos\theta, y = r\sin\theta$. By using the Chain Rule prove that compute $\frac{\partial z}{\partial r} = 2r$ and $\frac{\partial z}{\partial \theta} = 0$.