

Spring 2009: Math 241 (Section 02); Practice EXAM 2 (Bis)

1. Let the position of an object in motion be traced out by

$$\vec{r}(t) = e^t \cos t \vec{i} + e^t \sin t \vec{j} + e^t \vec{k}.$$

- (a.) Find the velocity and the acceleration of the object.
(d.) What is the radius of curvature at the point $\vec{r}(1)$?

2. Let $z = x^{-1}e^{-y/x}$ for $x \neq 0$. Show that z satisfies the equation

$$\frac{\partial z}{\partial x} = y \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y}.$$

3. Consider the function $f(x, y, z) = xye^z$.

- (a.) Find the direction in which f increases most rapidly at the point $(2, 3, 0)$. What is the maximum directional derivative of f at this point?
(b.) Let S be the surface described by the equation $f(x, y, z) = 6$ where $f(x, y, z)$ is given above. Find an equation for the plane tangent to S at the point $(2, 3, 0)$.

4. Let $w = f(u, v, t)$, where $u = y - x$, $v = y - z$, $t = z - x$ and f is a differentiable function. Compute the sum

$$A = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}.$$