Spring 2009: Math 241 (Section 02); Practice EXAM 4

1. Let $\vec{h}(x,y) = (2xy - y^2)\,\vec{\imath} + (x^2 - 2xy)\,\vec{\jmath}$ be a vector field.
(a.) Is $\vec{h}$ a conservative vector field? If no justify your answer, if yes find a function f whose gradient is $\vec{h}$.
(b.) Using the Fundamental theorem of Line Integral and part a. compute the line integral of $\vec{h}$ over the curve $\vec{r}(t) = \cos t^2\,\vec{\imath} + \sin\, t^2\vec{\jmath}, \quad 0 \le t \le \sqrt{\frac{\pi}{2}}$.
(c.) Compute the line integral of $\vec{h}$ over the curve $\vec{r}(t) = (1-t)\,\vec{\imath} + t\,\vec{\jmath}, \quad 0 \le t \le 1$ by direct integration.
(d.) Do your results from a. and b. agree? Explain why or why not.

2. Let R be the region of the plane bounded by the lines $x = 1$ and $x = 2$ and the parabolas $y - x^2 = 0$ and $y - x^2 = 3$. Compute the following double integral

$$\iint_R (xy - x^3)\,dA$$

by using the following change of variables: $u = x$, and $v = y - x^2$.

3. Let S be the surface given by $(x-z)^2 + 2y^2 = 4,\ 0 \le z \le 4$.
(a.) Verify that $\vec{r}(u,v) = (v + 2\cos u)\,\vec{\imath} + \sqrt{2}\,\sin u\,\vec{\jmath} + v\,\vec{k},\ 0 \le u \le 2\pi, 0 \le v \le 4$ is a parametrization of the surface S.
(b.) Find the area of the surface S.

4. Express and evaluate the following integral in spherical coordinates:

$$\iiint_D \sqrt{(x^2+y^2+z^2)}e^{-(x^2+y^2+z^2)}\,dV,$$

where the solid region D is the ball of center 0 and radius r.