Spring 2009: Math 241 (Section 02); Practice EXAM 4

1. Consider the vector field

$$\vec{F}(x, y, z) = yz^2\,\vec{\imath} + xz^2\,\vec{\jmath} + [2xyz + 2\cos(2z)]\vec{k}.$$

(a.) Prove that $\vec{F}$ is a conservative vector field and find a potential function f for $\vec{F}$.
(b.) (b.) Determine $curl\vec{F}$.
(c.) Evaluate the following line integral

$$I = \int_C \vec{F} \cdot d\vec{r},$$

where C is the oriented curve parametrized by $\vec{r}(t) = \cos(\pi t^9)\vec{\imath} + t^{10}\vec{\jmath} + \frac{\pi}{2}\frac{t}{1+t^4}\vec{k}$ and $-1 \leq t \leq 1$.

2. Let R be the region of the plane bounded by the curves $xy = \pi/2$ and $xy = \pi$, $y(2-x) = 2$ and $y(2-x) = 4$. Compute the following double integral

$$\iint_R y\cos(xy)dA,$$

by using the following change of variables: $x = \frac{2v}{u+v}$, and $y = u + v$.

3. Let S be the portion of the surface $z = xy$ that is inside the cylinder $x^2 + y^2 = 1$.
(a.) Find a parametrization of S.
(b.) Find the area of the surface S.

4. Express and evaluate the following integral in cylindrical coordinates:

$$\iiint_D (z^2 + 1)\,dV,$$

where D is the solid region bounded below by the upper nappe of the cone $z^2 = 3x^2 + 3y^2$ and above by the sphere $x^2 + y^2 + z^2 = 4$.