

Math 464

Homework: Due on 4/9

1) Consider the following boundary value problem (this is the heat equation with Neumann boundary conditions).

$$\begin{cases} u_t = u_{xx} & : t > 0, 0 \leq x \leq \pi \\ u(x, 0) = f(x) = x^2 - x & : 0 \leq x \leq \pi \\ u_x(0, t) = u_x(\pi, t) = 0. & : \end{cases}$$

Where  $u_t = \frac{\partial u}{\partial t}$ ,  $u_x = \frac{\partial u}{\partial x}$ , and  $u_{xx} = \frac{\partial^2 u}{\partial x^2}$ . Note the solution  $u(x, t)$  of this problem represents the temperature of a rod of length  $\pi$  at position  $x$  and at time  $t$ , with initial temperature  $f(x) = x^2 - x$  and insulated at its end points.

a) Show that the solution of the previous boundary value problem is given by

$$u(x, t) = \sum_{k=0}^{\infty} a_k e^{-k^2 t} \cos(kx)$$

for some coefficients  $a_k$  to be specified.

b) In which sense(s) is the last series converging?

c) What can you say about the rate of convergence of the last series? Do we need to take a lot of terms to have a good approximation using partial sums? What is the long term temperature of the rod?

2) Find the laplace transform of each of the following functions:

$$f(t) = \cos 4t, f(t) = 2t^{3/2}, f(t) = t^2 e^{-t}.$$

3) Find the Laplace transform of the derivative of each function in problem 2.