Sample Problems for Second In-Class Exam
Math 246, Fall 2009, Professor David Levermore

(1) Give the interval of existence for the solution of the initial-value problem

$$\frac{\mathrm{d}^3x}{\mathrm{d}t^3} + \frac{\cos(3t)}{4-t}\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{e^{-2t}}{1+t}, \qquad x(2) = x'(2) = x''(2) = 0\,.$$

(2) Suppose that $Y_1(t)$ and $Y_2(t)$ are solutions of the differential equation

$$y'' + 2y' + (1+t^2)y = 0\,.$$

Suppose you know that $W[Y_1, Y_2](0) = 5$. What is $W[Y_1, Y_2](t)$?

(3) Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-2+i3$, $-2-i3$, $i7$, $i7$, $-i7$, $-i7$, 5, 5, 5, -3, 0, 0.
(a) Give the order of L.
(b) Give a general real solution of the homogeneous equation $\mathrm{L}y = 0$.

(4) Let $\mathrm{D} = \dfrac{\mathrm{d}}{\mathrm{d}t}$. Solve each of the following initial-value problems.

(a) $\mathrm{D}^2y + 4\mathrm{D}y + 4y = 0\,, \qquad y(0) = 1\,, \quad y'(0) = 0\,.$

(b) $\mathrm{D}^2y + 9y = 20e^t\,, \qquad y(0) = 0\,, \quad y'(0) = 0\,.$

(5) Let $\mathrm{D} = \dfrac{\mathrm{d}}{\mathrm{d}t}$. Give a general real solution for each of the following equations.

(a) $\mathrm{D}^2y + 4\mathrm{D}y + 5y = 3\cos(2t)\,.$

(b) $\mathrm{D}^2y - y = t\,e^t\,.$

(c) $\mathrm{D}^2y - y = \dfrac{1}{1+e^t}\,.$

(6) Let $\mathrm{D} = \dfrac{\mathrm{d}}{\mathrm{d}t}$. Consider the equation

$$\mathrm{L}y = \mathrm{D}^2y - 6\mathrm{D}y + 25y = e^{t^2}\,.$$

(a) Compute the Green function $g(t)$ associated with L.
(b) Use the Green function to express a particular solution $Y_P(t)$ in terms of definite integrals.

(7) The functions t and t^2 are solutions of the homogeneous equation

$$t^2\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 2t\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = 0 \qquad \text{over } t > 0\,.$$

(You do not have to check that this is true!)

(a) Compute their Wronskian.

(b) Solve the initial-value problem

$$t^2\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 2t\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = t^3e^t\,, \qquad y(1) = y'(1) = 0\,, \qquad \text{over } t > 0\,.$$

Try to evaluate all definite integrals explicitly.

(8) What answer will be produced by the following MATLAB commands?

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>> ode1 = 'D2y + 2*Dy + 5*y = 16*exp(t)';
>> dsolve(ode1, 't')
ans =
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(9) The vertical displacement of a mass on a spring is given by

$$h(t) = 4e^{-t}\cos(7t) - 3e^{-t}\sin(7t)\,.$$

(a) Express $h(t)$ in the form $h(t) = Ae^{-t}\cos(\omega t - \delta)$ with $A > 0$ and $0 \le \delta < 2\pi$, identifying the quasiperiod and phase of the oscillation. (The phase may be expressed in terms of an inverse trig function.)

(b) Sketch the solution over $0 \le t \le 2$.

(10) When a mass of 4 grams is hung vertically from a spring, at rest it stretches the spring 9.8 cm. (Gravitational acceleration is $g = 980$ cm/sec^2.) At $t = 0$ the mass is displaced 3 cm above its equilibrium position and is released with no initial velocity. It moves in a medium that imparts a drag force of 2 dynes (1 dyne = 1 gram cm/sec^2) when the speed of the mass is 4 cm/sec. There are no other forces. (Assume that the spring force is proportional to displacement and that the drag force is proportional to velocity.)

(a) Formulate an initial-value problem that governs the motion of the mass for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)

(b) What is the natural frequency of the spring?

(c) Show that the system is under damped and find its quasifrequency.