Quiz 2 Solutions, Math 246, Professor David Levermore Tuesday, 15 September 2009

(1) [4] Sketch the phase-line portrait for the equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -w^2(w-3)(w-6)\,, \qquad w(0) = w_o\,.$$

Identify each stationary (equilibrium) point as either stable, unstable, or semistable. (You do not have to find the solution!)

Solution: The stationary points are w = 0, w = 3, and w = 6. The phase-line is



(2) [4] In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population and its population would double every three weeks. There are 120,000 mosquitoes in the area initially, and predators eat 90,000 mosquitoes per week. Write down an initial-value problem that governs the population of mosquitoes in the area at any time. (You do not have to solve the initial-value problem!)

Solution: Let M(t) be the number of mosquitoes at time t weeks. Doubling every three weeks corresponds to a growth factor of $2^{\frac{t}{3}} = (e^{\log(2)})^{\frac{t}{3}} = e^{\frac{1}{3}\log(2)t}$, which implies a growth rate of $\frac{1}{3}\log(2)$. The initial-value problem that M satisfies is therefore

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{1}{3}\log(2)M - 90,000\,,\qquad M(0) = 120,000$$

(3) [2] Consider the initial-value problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} + 5t^4 e^{-y} = 0, \qquad y(0) = y_o \quad \text{for some } y_o \text{ in } (-\infty, \infty)$$

Its solution satisfies

$$y = -t^5 + e^{y_o} \,.$$

Give its interval of existence as a function of y_o .

Solution: The interval of existence is $(-\infty, e^{y_o/5})$ because one needs

$$-t^5 + e^{y_o} > 0 \,,$$

which implies $t^5 < e^{y_o}$, which implies $t < e^{y_o/5}$.