Quiz 3 Solutions, Math 246, Professor David Levermore
Tuesday, 22 September 2009

(1) [4] Consider the following MATLAB function M-file.

```
function [t,y] = solveit(ti, yi, tf, n)

h = (tf - ti)/n;
t = zeros(n + 1, 1);
y = zeros(n + 1, 1);
t(1) = ti;
y(1) = yi;
for j = 1:n
t(j + 1) = t(j) + h;
s = t(j) + h/2;
x = y(j) + h*(t(j)^2 + y(j)^4)/2;
y(j + 1) = y(j) + h*(s^2 + x^4);
end
```

(a) What initial-value problem is being approximated numerically?
(b) What numerical method is being used?

Solution: The initial-value problem being approximated is

$$\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 + y^4 \,, \qquad y(t_i) = y_i \,.$$

The Heun-Midpoint method is being used.

(2) [2] Suppose you are using the Heun-Trapezoidal method to numerically approximate the solution of an initial-value problem over the time interval $[0, 4]$. By what factor would you expect the global error to decrease when the number of time steps that you take is increased from 400 to 1200.

Solution: When you increase the number of time steps by a factor of 3, the time step h is reduced by a factor of 3. Because the Heun-Trapezoidal method is second order, the global error will therefore decrease by a factor of $3^2 = 9$.

(3) [4] Find an implicit general solution of the exact differential form

$$(2x - y)dx + (2y - x)dy = 0 \,.$$

Solution: Because this differential form is exact, we can find $H(x, y)$ such that

$$\partial_x H(x, y) = 2x - y \,, \qquad \partial_y H(x, y) = 2y - x \,.$$

Upon integrating the first equation with respect to x you find

$$H(x, y) = \int 2x - y\mathrm{d}x = x^2 - xy + h(y) \,.$$

When this is plugged into the left-hand side of the second equation you obtain $-x + h'(y) = 2y - x$, which yields $h'(y) = 2y$. By taking $h(y) = y^2$, a general solution is then given by

$$H(x, y) = x^2 - xy + y^2 = c \,.$$