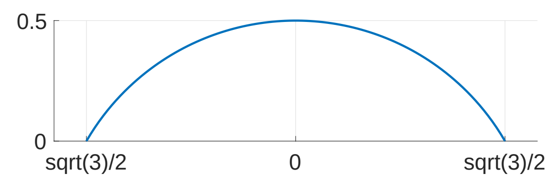
AMSC 466. Spring 2020

Homework Assignment 8. Due Thursday April 16.

1. **(5 pts)** Let $f(x)$ be an arc of the unit circle centered at $(0, -1/2)$ of angle $2\pi/3$:

$$f(x) = \sqrt{1-x^2} - 1/2, \quad [-\sqrt{3}/2, \sqrt{3}/2]\,.$$

The graph of $f(x)$ is shown in the figure below.



   (a) Calculate and write out explicitly the Hermite interpolation polynomial $p_5(x)$ with the abscissas $t_0 = t_1 = -\sqrt{3}/2$, $t_2 = 0$, $t_3 = t_4 = \sqrt{3}/2$.

   (b) Plot the graphs of $f(x)$ and $p_5(x)$ in the same figure. Find the exact maximal interpolation error. *Hint: you can use Matlab's function* `fzero` *or whatever you find appropriate for finding the point at which the difference between $f(x)$ and $p_5(x)$ is maximal in absolute value.*

2. **(5 pts)** Take the function "Witch of Agnesi" $f(x) = (1+x^2)^{-1}$ on the interval $[-5, 5]$, take the Chebyshev-Gauss-Lobatto nodes

$$x_k = 5\cos\left(\frac{\pi k}{n}\right), \quad k = 0, 1, \ldots, n,$$

and write a program computing Newton's interpolation polynomial for $n = 4, 8, 12, 16$. Plot the graph of $f(x)$ together with the interpolants in the same figure. For each n, estimate the maximal interpolation error. *Hint: mimic the program in Section 2.4 of* `interpolation.pdf`.

3. **(8 pts)** Prove properties (A), (B), (F), and (G) of Chebyshev's polynomials in Section 3.1 of `interpolation.pdf`.