Homework 6. Due Thursday, Dec. 3

1. **(5 pts)** Implement the DFS to check experimentally the theoretical result for the Poisson random graph with mean degree $z = p(n-1)$.

   - The fraction S of vertices in the giant component is the largest solution of
   $$S = 1 - \exp(-zS);$$
   - Let v be a randomly chosen vertex from a non-giant component. The average size of the component to which v belongs is
   $$\langle s \rangle = \frac{1}{1 - z + zS}.$$

   Proceed as follows. Program the DFS (from scratch). Set the number of vertices $n = 1000$. Define a grid of values of z ranging between 0 and 4. For each z, generate $r = 100$ random graphs $G(n,p)$ where $p = z/(n-1)$. For each graph, use the DFS to find its connected components. Calculate $\langle s(z) \rangle$ and $S(z)$. Make two figures:

   - Figure 1: find numerically and plot the theoretical values $S(z)$ versus z. Also plot the experimentally found values for $S(z)$.
   - Figure 2: plot the theoretical value for $\langle s(z) \rangle = [1 - z + zS(z)]^{-1}$ and the experimentally found values for $\langle s(z) \rangle$.

   Comment on your findings. Link files with your codes to the pdf file with your report.

2. **(5 pts)** Implement the BFS to obtain estimates for the average length of shortest paths in the Poisson random graph:
   $$l \simeq \frac{\log(n)}{\log(z)}. \tag{1}$$

   Proceed as follows. Program the BFS (from scratch). Set $z = 4$ so that almost all vertices belong to the giant component. For $n = 2^p$, $p = 10, 11, 12, 13$, generate a random graph $G(n,p)$. Randomly select $r = 100$ vertices, and use each of them as a seed for the BFS. Average the found path lengths and find $l(n)$. Plot the found $l(n)$ versus n as well as the theoretical estimate (1). Comment on your observations. Link files with your codes to the pdf file with your report.

3. **(5 pts)** Read Sections II.A, II.C, and II.D in Newman, Strogats, Watts, Random graphs with arbitrary degree distributions and their applications, Phys. Rev. E, 64, 026118. Consider a random graph with a prescribed degree distribution p_k. Let v be a randomly chosen vertex from a non-giant component. Write a book report with a detailed derivation of the formula for the average size of the component to which v belongs to:
   $$\langle s \rangle = 1 + \frac{zu^2}{[1 - S][1 - G_1'(u)]}. \tag{2}$$