Math 401 Syllabus, Fall 2009

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Office hour: Mondays and Wednesdays 12-1.

Topics: We will cover chapters 1-6 with strong emphasis on the applications, with additional material on the Fast Fourier Transform, baby Quantum Mechanics (up to Heisenberg’s Uncertainty Principle in finite dimensions) and linear codes (if time permits).

This is a second course in Linear Algebra, and you should already be familiar with the basic topics of most of these chapters. These will be reviewed quickly in class.

For exam purposes, you are responsible for the topics for which homework was assigned or notes were distributed.

Grading:
2 in-class exams 25% each
Homework 10%
Final exam: 40%

Students with a total score of less than 50% will fail the course.

Make-up policy: No make-ups for in-class exams. In the case of an absence due to illness, religious observance, participation in a University activity at the request of University authorities, or other compelling circumstances, the missed in-class exam will be replaced with the average of the other exams (in-class and final). No late homeworks, but I will drop the lowest two homework grades.

Problem set 1, due Wednesday, September 9.
1.7: 3, 4, 5

Revised assignment

Problem set 2, due Friday September 25.

Work through section 2.5 until you understand it. Solve 2.5 :6

Also solve: An electrical circuit is built on the graph in figure 2.6. There is one battery of strength 9 on edge 1 \(b_1 = 9\), and edge 1 has resistance 0. Edges 2, 3 and 4 have resistance 1 each and edge 5 has
resistance 0. There is a current “source” $f_1 = -5$ at node 1 which has an on/off switch, and node 4 is grounded.

Set up the matrix equation associated to this problem. This is similar to Example 1, page 121, with the same sign conventions.

Solve the equation and find the currents $y_1, \cdots y_5$. Do this first with the switch off, then with the switch on. Before computing, do you expect the magnitude of current in edge 1 to increase when you turn the switch on? What about edge 2? Any method is acceptable, and Matlab is strongly recommended.

Problem set 3, due Monday October 12.

3.5: 1, 4, 12

Notes: Baby introduction to time-independent Quantum Mechanics

This refers to a quantum system (such as an atom) and observable quantities, such as energy.

These aren’t usually ”observed” deterministically, nor are all possible number eligible as outcomes of the observation. Rather, to each observable one associates a Hermitian linear transformation $A$ (think of $A$ as a Hermitian matrix). The only allowed (observed) values of that observable are the eigenvalues of $A$: $\lambda_1, \lambda_2 \cdots$ (in the set-up of quantum mechanics, things happen in infinite dimensions).

Let the corresponding orthonormal (why orthonormal?) eigenvectors of $A$ be $v_1, v_2, \cdots$.

The state of the quantum system is described by a complex vector $v$ of norm 1,

$$v = c_1 v_1 + c_2 v_2 + \cdots$$

$c_i$ are complex numbers and $|c_1|^2 + |c_2|^2 + \cdots = 1$ (why?).

When the system is in this state $v$, the value $\lambda_i$ is observed with probability $|c_i|^2$.

If $v = v_i$, the observable in question is for sure (deterministically) equal to $\lambda_i$, but in general the result of an observation is probabilistic.

The expected value of the observable $A$ in the state $v$ is defined as

$$< A >_v = \lambda_1 |c_1|^2 + \lambda_2 |c_2|^2 + \cdots = v^H A v \text{ (in the language of Math401)}$$

The first equality above is a definition. Why is the second equality true?

The uncertainty of $< A >_v$ (when the system is in the state $v$) is defined as the square root of the expected value of $(A - < A >_v I)^2$, or
\[ \Delta A_v = (v^H (A - < A >_v I)^2 v)^{1/2} \]
\[ = \| (A - < A >_v I) v \| \]

Again, the first equality above is a definition. Why is the second equality true?

Exercise Check that the uncertainty is zero if \( v = v_i \) is an eigenvector.

In practice, one wishes for observations with small uncertainty.

The commutator of two matrices is \([A, B] = AB - BA\).

Can one simultaneously measure two observables \( A, B \) with small uncertainty? If \( A \) and \( B \) commute \(([A, B] = 0)\), they have the same eigenvectors (proved in class), and the answer is yes. If \( A \) and \( B \) don’t commute, the negative answer is a famous result:

Heisenberg’s uncertainty principle If two Hermitian matrices \( A, B \) satisfy

\[ [A, B] = C \]

then

\[ \Delta A_v \Delta B_v \geq \frac{1}{2} |v^H Cv| \]

This explains, for instance, why the position and momentum of a quantum particle cannot both be precisely measured at the same time: their commutator \( C \) is (a constant times) the identity.

Exercise Show that for no matrices \( A \) and \( B \) can \([A, B] = I\).

However, this can happen for the infinite dimensional “matrices” corresponding to position and momentum.

Any of the mathematical issues on this handout are fair exam questions.

Homework 4, due Monday, Nov. 2:
NAME:
Write down short ”proofs” for the following mathematical results needed to make sense of the previous discussion. All but the last one were discussed in class.

1) If \( A \) is Hermitian, its eigenvalues are real.

2) If \( A \) is Hermitian, the eigenvectors corresponding to distinct eigenvectors are orthogonal \((v^H w = 0)\).

3) For any square matrix \( A \), Hermitian or not, it is possible to find \( T \) upper triangular and \( U \) unitary \((U^H U = I)\) such that \( A = UTU^H \). Prove this in the 2x2 case.
4) If $A$ is Hermitian, then $T$ from the previous problem is diagonal and $A$ has a basis of orthonormal eigenvectors $v_1, \ldots, v_n$.

5) If $v_1, \ldots, v_n$ are an orthonormal basis, and if $v = c_1 v_1 + \cdots + c_n v_n$ and $w = d_1 v_1 + \cdots + d_n v_n$, show $v^H w = \overline{c_1 d_1} + \cdots + \overline{c_n d_n}$.

6) If $v_1, \ldots, v_n$ are an orthonormal basis, and if $v = c_1 v_1 + \cdots + c_n v_n$ and $A$ is Hermitian, prove

$$\left( v^H (A - \langle A \rangle_I) v \right)^{1/2} = \| (A - \langle A \rangle_I) v \|$$. 

Tentative dates for the in-class exams: Friday, October 16 and Wednesday, November 18. The final exam will be on Wednesday, Dec 16 8:00am-10:00am.