

**Fall 2008 - Math 462 Section 0101**  
**Partial Differential Equations for Scientists and Engineers**  
Homework #11 - Due Tuesday Nov 25th

1. (20pt) For each of the following functions, state whether it is even or odd or periodic. If periodic, what is the smallest period?
  - (a)  $\tan(ax)$  ( $a > 0$ )
  - (b)  $\sin(x^2)$
  - (c)  $e^{-x}$
  - (d)  $x^m$  ( $m = \text{integer}$ )
  - (e)  $\frac{x^2 \sin(x)}{2 + \cos(x)}$
  
2. (30) Compute the full Fourier series of the function  $f(x) = |x|$  on the interval  $(-L, L)$ . Sketch the function to which the Fourier series converges on the interval  $(-3L, 3L)$ .
  
3. (25pt)
  - (a) Let  $\phi(x)$  be a continuous function on  $(0, L)$ . Under what conditions is its **odd** extension also a continuous function?
  - (b) Same as part (a) for the **even** extension.
  
4. (25pt) We saw in Homework #9 that the boundary conditions  $X(0) = 0$ ,  $X'(1) = 0$  lead to the eigenfunctions

$$X_n(x) = \sin\left(\left(n + \frac{1}{2}\right)\pi x\right) \quad n = 0, 1, 2, \dots$$

Assuming that a function  $\phi(x)$  defined on  $(0, 1)$  can be written as

$$\phi(x) = \sum_{n=0}^{\infty} c_n \sin\left(\left(n + \frac{1}{2}\right)\pi x\right)$$

find a formula for the coefficient  $c_n$  as an integral of  $\phi$ .