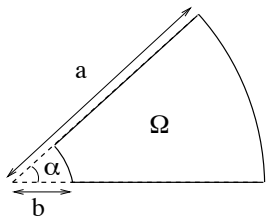
Fall 2008 - Math 462 Section 0101
Partial Differential Equations for Scientists and Engineers
Homework #13 - Not due

1. Consider a domain Ω obtained by taking a circular sector with angle α and radius a and cutting out a smaller circular sector of radius b:



Find the solution of the following BVP in Ω:

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ u(r,0) = 0 \quad \text{for } b < r < a, & u(r,\alpha) = 0 \quad \text{for } b < r < a \\ u(b,\theta) = 0 \quad \text{for } 0 < \theta < \alpha, & u(a,\theta) = f(\theta) \quad \text{for } 0 < \theta < \alpha \end{cases}$$

2. Find the solution of Laplace's equation in the semi-infinite strip $\{(x,y)\,;\, 0 \le x \le 2\,,\ y \ge 0\}$ satisfying the following mixed boundary conditions:

$$\begin{cases} u(0,y) = 0, \qquad u_x(2,y) = 0 \quad \text{for all } y \ge 0 \\ u(x,0) = 2\sin(3\pi x/4) - 3\sin(7\pi x/4) \quad \text{for all } 0 \le x \le 2, \\ \lim_{y\to+\infty} u(x,y) = 0 \end{cases}$$

3. Solve the following BVP in the rectangle $\{(x,y)\,;\, 0 \le x \le 1\,,\ 0 \le y \le 1\}$:

$$\begin{cases} \Delta u = 0 \qquad 0 \le x \le 1\,,\ 0 \le y \le 1 \\ u_x(0,y) = 0 \quad 0 \le y \le 1, & u_x(1,y) = \cos(\pi y) + \cos(2\pi y) \quad 0 \le y \le 1 \\ u_y(x,0) = 0 \quad 0 \le x \le 1, & u_y(x,1) = 0 \quad 0 \le x \le 1 \end{cases}$$