

Fall 2008 - Math 462 Section 0101
Partial Differential Equations for Scientists and Engineers
Homework #3 - Due Thursday Sept 25th

1. (30 pts) The midpoint of a piano string of tension T , density ρ and length ℓ is hit by a hammer whose head diameter is $2a$. A flea is sitting at a distance $\ell/4$ from one end. How long does it take for the disturbance to reach the flea? (assuming $a < \ell/4$).

2. (30 pts) Consider $u(x, t)$ solution of the IVP

$$\begin{aligned}u_{tt} - c^2 u_{xx} &= 0 && \text{for } -\infty < x < +\infty, t > 0 \\u(x, 0) &= 0 && \text{for all } x \\u_t(x, 0) &= \psi(x) && \text{for all } x\end{aligned}$$

where

$$\psi(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

Sketch u for $t = 0$, $t = a/2c$, $t = a/c$, $t = 3a/2c$, $t = 2a/c$ and $t = 5a/c$.

3. (40 pts) Solve

$$u_{xx} - 3u_{xt} - 4u_{tt} = 0$$

with initial conditions

$$u(x, 0) = x^2, \quad u_t(x, 0) = e^x.$$

(Hint: Factor the operator as we did for the wave equation.)