

Fall 2008 - Math 462 Section 0101
Partial Differential Equations for Scientists and Engineers
Homework #4 - Due Thursday Oct 2nd

1. (30pt) Consider the damped string equation (see Homework 2):

$$u_{tt} - c^2 u_{xx} + \gamma u_t = 0.$$

Show that the total energy decreases.

2. (40pt)

- (a) If $u_t - u_{xx} = f$ and $v_t - v_{xx} = g$ with $f \leq g$ and $u \leq v$ at $x = 0$, $x = L$ and $t = 0$, prove that $u \leq v$ for $x \in [0, L]$, $t > 0$.
- (b) If $v_t - v_{xx} \geq \sin x$ for $0 \leq x \leq \pi$ and $t > 0$, and if $v(0, t) \geq 0$, $v(\pi, t) \geq 0$ and $v(x, 0) \geq \sin x$, use part (a) to show that

$$v(x, t) \geq (1 - e^{-t}) \sin x$$

3. (30pt) Consider the diffusion equation on $(0, L)$ with Robin boundary conditions $u_x(0, t) - a_0 u(0, t) = 0$ and $u_x(L, t) + a_L u(L, t) = 0$. If $a_0 > 0$ and $a_L > 0$, use the energy method to show that the endpoints contribute to the decrease of the energy $\int_0^L u^2(x, t) dx$.