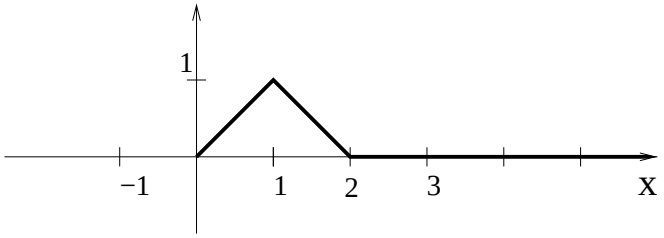
Fall 2008 - Math 462 Section 0101
Partial Differential Equations for Scientists and Engineers
Homework #7 - Due Thursday Oct 23th

1. (25pt) Let $u(x,t)$ be the solution of the wave equation on the half-line with Neumann boundary condition:

$$\begin{array}{ll} u_{tt} - u_{xx} = 0 & 0 < x < \infty,\ \ t > 0, \\ u(x,0) = \phi(x), & u_t(x,0) = 0, \\ u_x(0,t) = 0 & \end{array}$$

with ϕ given by:



Sketch $u(x,t)$ for $t = 1/2$, $t = 1$, $t = 2$, and describe the behavior of u as t increases.

2. (25pt) Let $u(x,t)$ be the solution of

$$\begin{array}{ll} u_{tt} - 4u_{xx} = 0 & 0 < x < \infty,\ \ t > 0 \\ u(x,0) = \phi(x), & u_t(x,0) = \psi(x), \\ u(0,t) = 0 & \end{array}$$

with ϕ given as in the problem 1 above, and $\psi = \begin{cases} 1 & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$.

Using the domain of influence, determine

   (a) The time at which $u(10,t)$ becomes non zero for the first time.
   (b) The time after which you are sure that $u(10,t)$ will always be zero.

You do not have to solve the PDE.

3. (25pt) Using Problem #4 in Homework #6, sketch the domain of influence of a point $x = a,\ t = 0$ in the $xt-$plane for the wave equation on the half-line with **Neumann** boundary condition. Then do the same for an interval $x \in (a,b),\ t = 0$.

4. (25pt) Solve the following inhomogeneous diffusion equation on the half-line

$$\begin{aligned} &u_t - k u_{xx} = f(x,t) \quad 0 < x < \infty, \quad 0 < t < \infty \\ &u(x,0) = \phi(x) \\ &u(0,t) = h(t) \end{aligned}$$

by carrying out the substraction method begun in class.