Fall 2008 - Math 462 Section 0101
Partial Differential Equations for Scientists and Engineers
Homework #8 - Due Thursday Oct 30th

1. (30pt) Solve

$$\begin{aligned} &u_{tt} - c^2 u_{xx} = e^{ax} \qquad -\infty < x < \infty, \ \ t > 0, \\ &u(x,0) = 0, \\ &u_t(x,0) = \cos(x). \end{aligned}$$

2. (35pt) Let $f(x,t)$ be any function and let

$$u(x,t) = \frac{1}{2c} \iint_{\Delta} f$$

where Δ is the triangle of dependence. Verify directly by differentiation that

$$u_{tt} - c^2 u_{xx} = f, \qquad u(x,0) = 0, \quad u_t(x,0) = 0.$$

Hint: Use the fact that

$$u(x,t) = \frac{1}{2c} \int_0^t \int_{x-ct+cs}^{x+ct-cs} f(y,s)\, dy\, ds$$

and carefully use the chain rule to compute u_{tt} and u_{xx}.

3. (35pt) Find the values of λ for which the following boundary value problem has non trivial solutions:

$$X'' + \lambda X = 0 \quad \text{ for } 0 < x < L, \qquad X'(0) = 0, \quad X'(L) = 0.$$

For each such λ, find the corresponding solutions $X(x)$.