

Fall 2008 - Math 462 Section 0101
Partial Differential Equations for Scientists and Engineers
Homework #9 - Due Thursday Nov 6th

1. (25pt) Consider waves in a resistant medium that satisfy the problem

$$\begin{aligned}u_{tt} - c^2 u_{xx} &= -ru_t & 0 < x < L, \quad t > 0 \\u(0, t) &= 0, \quad u(L, t) = 0\end{aligned}$$

where r is a constant $0 < r < 2\pi c/L$. Use the separation of variables method to find a series solution of this boundary value problem.

2. (a) (25pt) Find the values of λ for which the following boundary value problem has non trivial solutions:

$$X'' + \lambda X = 0 \quad \text{for } 0 < x < 1, \quad X(0) = 0, \quad X'(1) = 0.$$

For each such λ , find the corresponding solutions $X(x)$.

- (b) (25pt) Use the separation of variables method and your answer to the question above to find the solution of the following **mixed** boundary problem:

$$\begin{aligned}u_t - k u_{xx} &= 0 & 0 < x < 1, \quad t > 0 \\u(0, t) &= 0, \quad u_x(1, t) = 0 \\u(x, 0) &= \sin\left(\frac{3\pi}{2}x\right) - 2\sin\left(\frac{5\pi}{2}x\right)\end{aligned}$$

3. (25pt) Find the eigenvalues graphically for the boundary problem

$$\begin{aligned}X'' + \lambda X &= 0 & 0 < x < L \\X(0) &= 0 & X'(L) + X(L) = 0\end{aligned}$$