

STAT 100 Lecture 2: Measures of Central Tendency

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Today's Agenda

- 1 Mathematical Notation
- 2 The Mean, Median, Percentiles, and Quantiles
- 3 Examples

Mathematical Notation

Sometimes, we have a lot of associated numerical values (either abstract or from a list) that we want to label in a similar manner. It is standard to write this list as x_1, x_2, x_3, \dots . If the number of numerical values is N , then we often write $\{x_i\}_{i=1}^N$ to denote the collection of this data.

Definition

*When we have a rather large sum of numbers, it is useful to employ **sigma notation**. Let $\{x_i\}_{i=1}^N$ be a collection of numbers. We set*

$$\sum_{i=1}^N x_i = x_1 + x_2 + \cdots + x_N$$

Definition of the Mean and the Median

Definition

The **Mean** of a set of N measurements $\{x_i\}_{i=1}^N$ is defined by

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

For example, the mean of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 100 is

$$\frac{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 100}{10} = 145/10 = 14.5.$$

Definition of the Median

Definition

The **Median** of a set of N measurements $\{x_i\}_{i=1}^N$ is defined to be middle value when the measurements are arranged from smallest to largest. If N is odd, there is a unique middle value. If N is even, we define the median to be the average of the two middle values.

For example, the median of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 100 is

$$\frac{5 + 6}{2} = 5.5.$$

Definition

Suppose $A = \{a_i\}_{i=1}^N$ is a set of numbers ordered from least to greatest, and further suppose that p is a number between 0 and 1. The $100p^{\text{th}}$ percentile of A is a number that is greater than $100 \times p$ percent of the numbers in A . The following steps are taken to calculate the $100p^{\text{th}}$ percentile of the set $\{x_i\}_{i=1}^N$:

- 1 Order the x_i 's from smallest to largest.
- 2 Calculate (sample size) \times (proportion) = Np
 - If Np is **not** an integer, round it up to the next integer and find that corresponding ordered value.
 - If Np is an integer (say k), average the k^{th} and $(k + 1)^{\text{th}}$ ordered values.

Quartiles

The quartiles are just the 25th, 50th, and 75th percentiles.

Exercise 2.30, 2.34, 2.47, 2.52

- Read Section 2.4 and 2.5 from Johnson and Bhattacharyya
- Online homework 2.4: 2.29, 2.33, 2.35, 2.53
 - Section 0101:
<http://edugen.wiley.com/edugen/class/cls73726/>
 - Section 0201:
<http://edugen.wiley.com/edugen/class/cls73727/>