

STAT 100 Lecture 6: Conditional Probability and Independence

Nate Strawn

<http://www.math.umd.edu/~nstrawn/>

① Definitions Galore

- Sample Space, Elementary Outcome, Event
- Discrete and Continuous Sample Spaces
- Probability, Properties of Probability

② Relations between Events

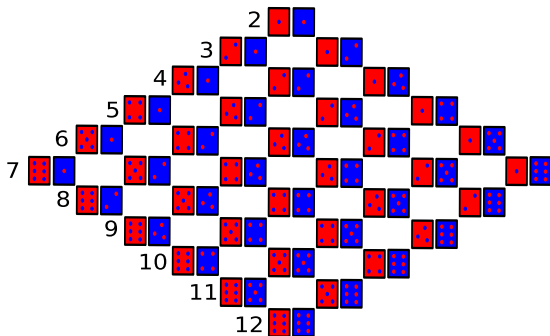
- Complements, Union, and Intersection
- Venn Diagrams!
- Law of Complements, Addition Law

Today's Agenda

- 1 Conditional Probability (Section 4.5)
- 2 Independence (Section 4.5)

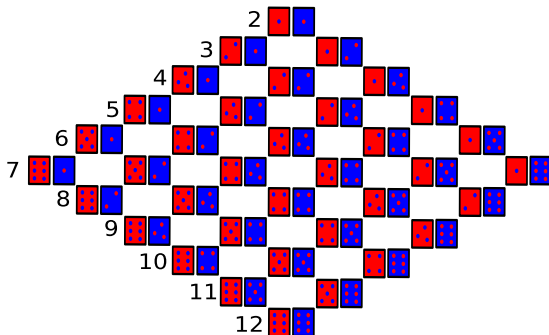
Motivating Conditional Probability

Sometimes we want to compute the probability of an event given some extra knowledge...



Motivating Conditional Probability

What is the probability of rolling above an 8 given that one of the dice has four dots?



Definition

The **Conditional Probability** of A given B is denoted $P(A|B)$ and defined by the formula

$$P(A|B) = \frac{P(AB)}{P(B)}.$$

Equivalently, we have

$$P(AB) = P(B)P(A|B).$$

This last formula is called the **Multiplication Law of Probability**.

Using Conditional Probability

Example

| | <i>Overweight</i> | <i>Normal</i> | <i>Underweight</i> | <i>Total</i> |
|-------------------------|-------------------|---------------|--------------------|--------------|
| <i>Hypertensive</i> | 0.10 | 0.08 | 0.02 | 0.20 |
| <i>Not Hypertensive</i> | 0.15 | 0.45 | 0.20 | 0.80 |
| <i>Total</i> | 0.25 | 0.53 | 0.22 | 1.00 |

- 1 *What is the probability that a person selected at random has hypertension?*
- 2 *If a person selected at random is overweight, what is the probability that the person is also hypertensive?*

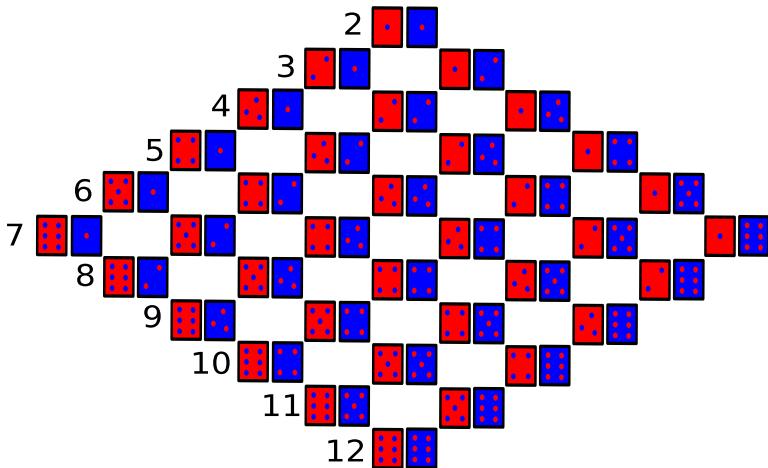
Using Conditional Probability

4.57 Suppose that $P(A) = 0.68$, $P(B) = 0.55$, and $P(AB) = 0.35$. Find

- (a) The conditional probability that B occurs given that A occurs.
- (b) The conditional probability that A occurs given that B occurs.
- (c) The conditional probability that B occurs given that A does not occur.

Sometimes, Extra Information Doesn't Matter

What is the probability that the blue die has 4 dots given that the red die has 2 dots?



Definition of Independence

Definition

Two events A and B are *Independent* if

$$P(A|B) = P(A).$$

Equivalently,

$$P(B|A) = P(B),$$

or

$$P(AB) = P(A)P(B).$$

Using Independence

4.66 Assume $P(A) = 0.4$, $P(B) = 0.4$, and $P(A \cup B) = 0.7$.

- (a) Are A and B independent? Why or why not?
- (b) Can A and B be mutually exclusive? Why or why not?

Independent Events are not Incompatible

Remember that incompatible events are such that AB is empty. So, we have $P(AB) = 0$. For independent events, we have $P(AB) = P(A)P(B)$.

Let $A =$ “The blue die has 6 dots”, $B =$ “The red die has 6 dots”, and $C =$ “The dots on both dice sum to 6”. What is $P(AB)$?
What is $P(AC)$?

For Next Time

- MINITAB Project 01 Due Next Wednesday! (Sept. 24).
- Read Section 4.5 and 4.6 from Johnson and Bhattacharyya
- Group Problems:

| | | | | | |
|---------|------|------|------|------|------|
| Group | 1 | 2 | 3 | 4 | 5 |
| Problem | 4.58 | 4.59 | 4.61 | 4.63 | 4.69 |
| Group | 6 | 7 | 8 | 9 | 10 |
| Problem | 4.58 | 4.59 | 4.61 | 4.63 | 4.69 |