STAT 100 Lecture 7: Random Samples

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Last Time...

1. Definition of Conditional Probability

$$P(A|B) = \frac{P(AB)}{P(B)}$$

2. Definition of Independence

$$P(A|B) = P(A)$$

or

$$P(AB) = P(A)P(B)$$

Today's Agenda

1. The Rule of Combinations (Section 4.6)
2. Definition of Random Sample (Section 4.6)
3. Examples (Section 4.6)

How Do We Count Combinations?

How many ways can we choose two people from a group of five?

How Do We Count Combinations?

This can become a very tedious exercsise!

Instead of writing out all of the pairs and counting, think of choosing a pair by putting the five people into chairs labeled 1 through 5. For the first chair, we have 5 choices. For the second, we've fixed the first chair, so we have 4 choices. Continuing this process gives us

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

choices. But we only care about pairings and not about orderings. How many different ways are there to arrage the people in the first two chairs? How about the last three chairs?

Another Way to Count

$$\text{number of pairs in a set of } 5 = \frac{5!}{2! \cdot 3!}$$

The Rule of Combinations

Definition

The number of possible choices of r objects from a group of N distinct objects is $\binom{N}{r}$ and is read "N choose r." We have that

$$\binom{N}{r} = \frac{N!}{r!(N-r)!}.$$

Formula

$$\binom{N}{r} = \frac{N \times (N-1) \times \cdots \times (N-r+1)}{r \times (r-1) \times \cdots \times 2 \times 1}$$

Counting Pairs by Formula

Example

The number of pairs that can be chosen from five distinct objects is

$$\binom{5}{2} = \frac{5 \times 4}{2 \times 1} = 20/2 = 10.$$

The number of pairs that can be chosen from six distinct objects?

$$\binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 30/2 = 15.$$

How about from seven distinct objects?

A Useful Identity

When we choose r objects from a set of N objects, we have implicitly chosen a collection of $N - r$ objects as well. These are the objects that we did not "choose." So, every choice of r objects corresponds to a choice of $N - r$ objects.

Proposition

$$\binom{N}{r} = \binom{N}{N-r}$$

Combinations Example

4.83 Evaluate:

(a) $\binom{6}{3}$

(b) $\binom{10}{3}$

(c) $\binom{22}{2}$

(d) $\binom{22}{20}$

(e) $\binom{30}{3}$

(f) $\binom{30}{27}$

Random Samples

Definition

A sample of size n from a population of N distinct objects is said to be a Random Sample if each collection of size n has the same probability of selection. This probability is exactly $1/\binom{N}{n}$. *Note that this means that we are using the uniform probability model on n-collections of N.*

Another Example

4.85 Of 10 available candidate for membership in a university committee, 6 are men and 4 are women. The committee is to consist of 4 persons.

(a) How many different selections of the committee are possible?

(b) How many selections are possible if the committee must have 2 men and 2 women?

(c) If the selection of the committe is random, what is the probability that the committee consists of exactly 2 men and 2 women?

For Next Time

- MINITAB Project 01 Due Today at 4pm!
- Read Section 5.1, 5.2, and 5.3 from Johnson and Bhattacharyya
- Group Problems:

Group	1	2	3	4	5
Problem	4.84	4.101	5.1	5.3	5.7
Group	6	7	8	9	10
Problem	4.84	4.101	5.1	5.3	5.7