STAT 100 Lecture 9: Mean and Standard Deviation of a Distribution

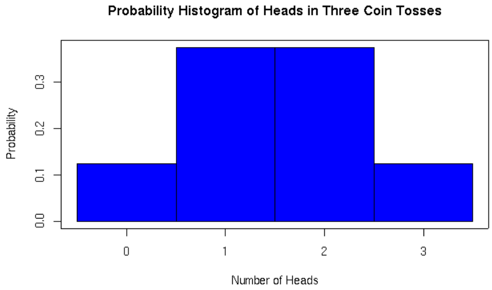
Nate Strawn

http://www.math.umd.edu/∼nstrawn/

Last Time...

1. Definition of Random Variable

2. Definition of Probability Distribution



Today's Agenda

1. Means of Distributions (Section 5.4)
2. Standard Deviations of Distributions (Section 5.4)
3. Examples

Another Way to Compute the Sample Mean

Ten students took a quiz and the following scores (from 0 to 5) resulted:

$$5, \quad 5, \quad 3, \quad 3, \quad , 5, \quad 4, \quad 4, \quad 0, \quad 2, \quad 4$$

The sample mean of this data is given by

$$\overline{x} = \frac{\text{Sum of Observations}}{\text{Sample Size}} = \frac{35}{10} = 3.5.$$

Another Way to Compute the Sample Mean

Using this data, we can also write down the empircal distribution of the random variable $X =$ Quiz Score:

x	$f(x) = P[X = x]$
0	0.10
1	0.00
2	0.10
3	0.20
4	0.30
5	0.30

Notice that we also have

$$\overline{x} = 0(0.10) + 1(0.00) + 2(0.10) + 3(0.20) + 4(0.30) + 5(0.30) = 3.5.$$

Mean of a Distribution

Definition

The Mean of random variable X (or Population Mean or Expected Value) is

$$E(X) = \mu = \sum (Value \times Probability) = \sum x_i f(x_i).$$

The mean of a random variable can be thought of as a "center of mass" of the distribution.

Using the formula for the mean, we see that the expected numer of heads in three coin tosses is

$$E(X) = 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8) = 12/8 = 1.5.$$

X	f(X)
0	1/8
1	3/8
2	3/8
3	1/8

Variance and Standard Deviation of a Random Variable

Definition

The *Variance* *and* *Standard Devation* *of a random variable X are (respectively):*

$$\sigma^2 = Var(X) = \sum (x_i - \mu)^2 f(x_i)$$

$$\sigma = sd(X) = \sqrt{Var(X)}$$

Formula

$$\sigma^2 = E(X^2) - E(X)^2 = E(X^2) - \mu^2 = \sum x_i^2 f(x_i) - \left(\sum x_i f(x_i)\right)^2$$

Back to the Coin Example

The expected number of Heads in three tosses of a coin is $E(X) = 1.5$. Now, let's compute the variance by first computing $E(X^2)$:

$$
\begin{aligned}
E(X^2) &= \sum x_i^2 f(x_i) \\
&= (0)^2\left(\frac{1}{8}\right) + (1)^2\left(\frac{3}{8}\right) + (2)^2\left(\frac{3}{8}\right) + (3)^2\left(\frac{1}{8}\right) \\
&= \frac{3+12+9}{8} = \frac{24}{8} = 3.
\end{aligned}
$$

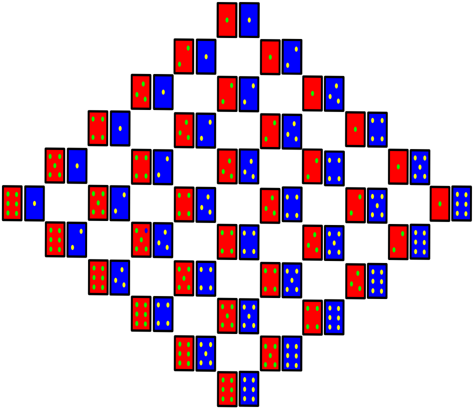
We now use the formula from the last slide to get

$$\sigma^2 = E(X^2) - E(X)^2 = 3 - (1.5)^2 = 3 - 2.25 = 0.75$$

and

$$\sigma = \sqrt{\sigma^2} = \sqrt{0.75} \approx 0.87.$$

Example: X = Sum of Two Dice



The Event "e"	X	f(X)
	2	1/36
	3	1/18
	4	1/12
	5	1/9
	6	5/36
	7	1/6
	8	5/36
	9	1/9
	10	1/12
	11	1/18
	12	1/36

For Next Time

- Review for Test on Friday!
- Group Problems:

Group	1	2	3	4	5
Problem	5.29	5.31	5.35	5.39	5.43
Group	6	7	8	9	10
Problem	5.29	5.31	5.35	5.39	5.43