STAT 100 Lecture 11: Continuous Distributions and the Normal Distribution

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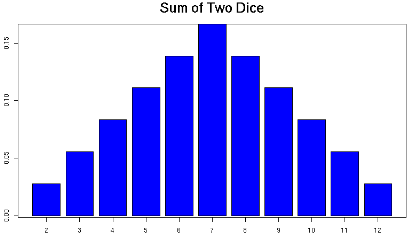
Last Time...

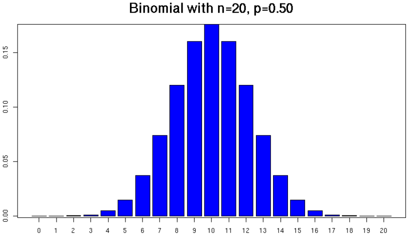
1. Bernoulli Trials
    - Two outcomes
    - Fixed success/failure rates p/q
    - Trials are independent.
2. The Binomial Distribution of parameters n and p
    - $\mu = np$
    - $\sigma^2 = np(1 - p)$
3. Using Binomial Tables

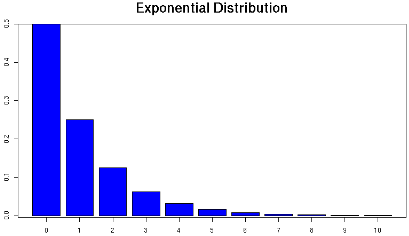
Today's Agenda

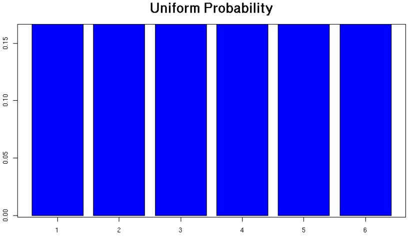
1. Continuous Distributions (Section 6.1)
2. The Normal Distribution (Section 6.2, 6.3)
3. Examples

Review of Relative Frequency Histograms









Probability Density Functions

Definition

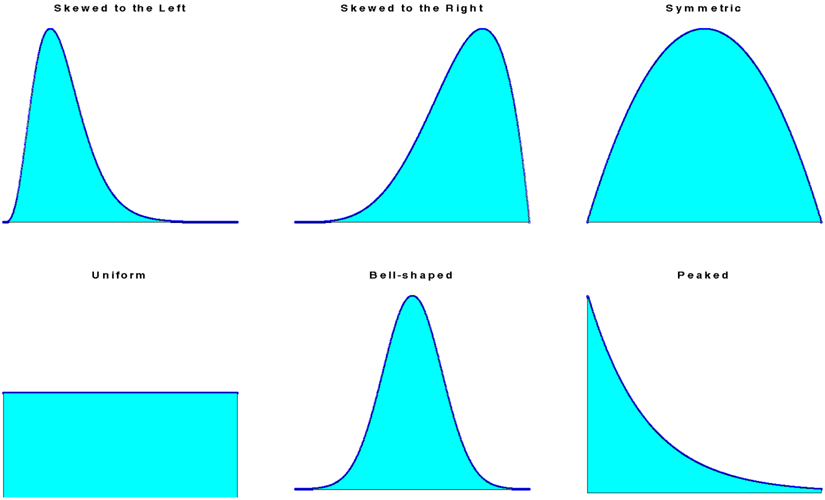
The *Probability Density Function* $f(x)$ *of a continuous random variable* X *is the function satisfying:*

1. *The total area under the* *Probability Density Curve* *is* 1.
2. $P[a \leq x \leq b] =$ *area between* a *and* b *under* $f(x)$.
3. $f(x) \geq 0$ *for all* x.

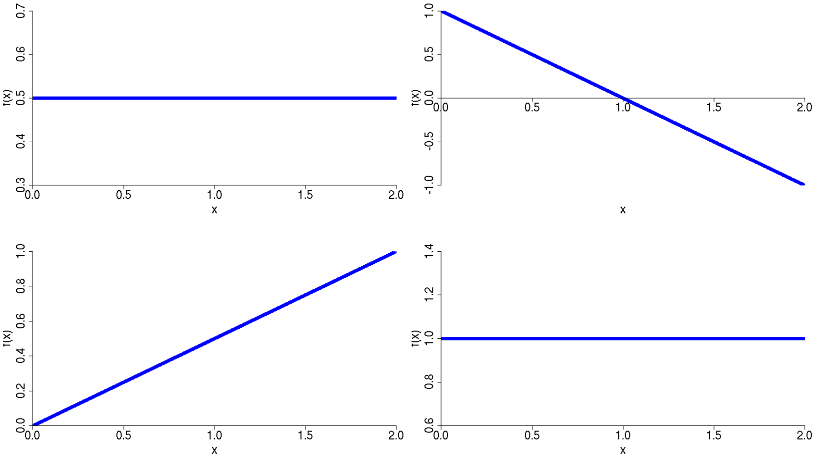
Note

By definition, we have that $P[X = x] = 0$ *for continuous random variables. Continuous probability only tells us the probability that a random variable lies in an interval.*

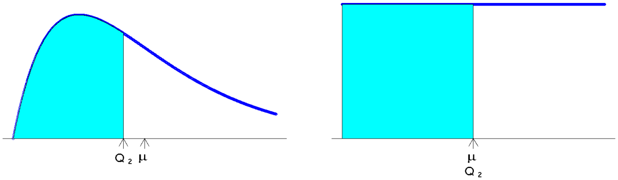
Properties of Continuous Distributions



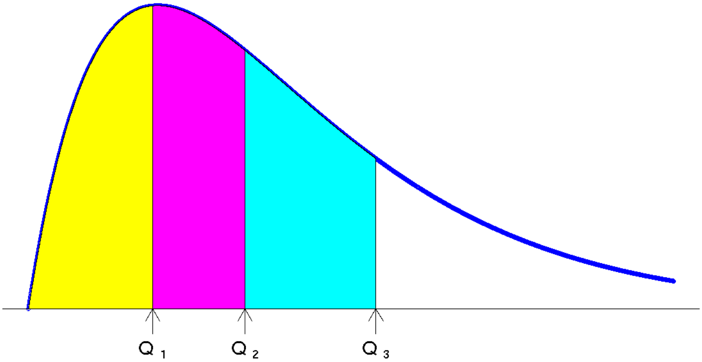
Am I a Probability Density Function?



For a continuous random variable, $E(X)$ is still the "center of gravity" of the distribution. The $100p^{th}$ percentile a the value x for which p is the area under the curve to the left of x and $1-p$ area under the curve lies to the right of x.



Quartiles of Continuous Random Variables



Standardized Variable

Definition

The Standardized Variable of the random variable X, with mean μ and standard devation σ. is

$$Z = \frac{\text{Variable} - \text{Mean}}{\text{Standard Deviation}} = \frac{X - \mu}{\sigma}.$$

Note that Z has mean 0 and standard deviation 1.

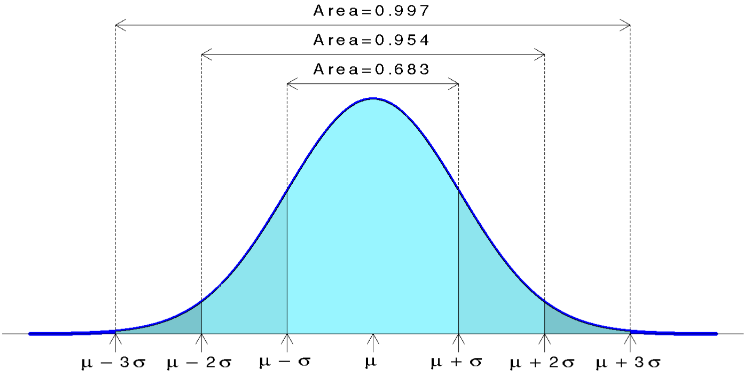
Example: Computing Standardized Variables

Example

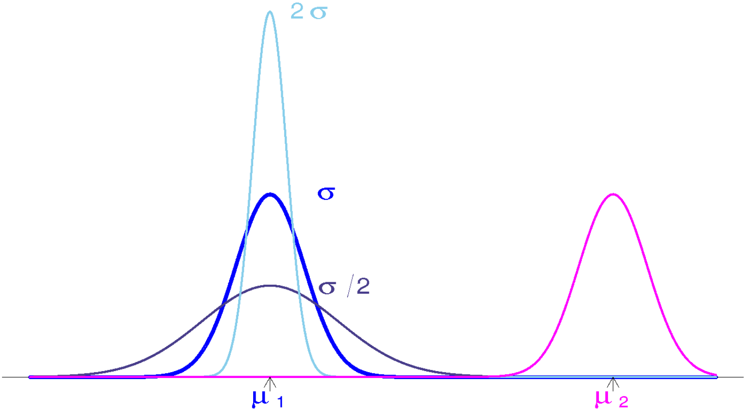
Compute the standardized variable Z if X is such that

- $\mu = 3$ *and* $\sigma = 12$
- $\mu = 40$ *and* $\sigma = 0.5$
- $\mu = 0.1$ *and* $\sigma = 9$

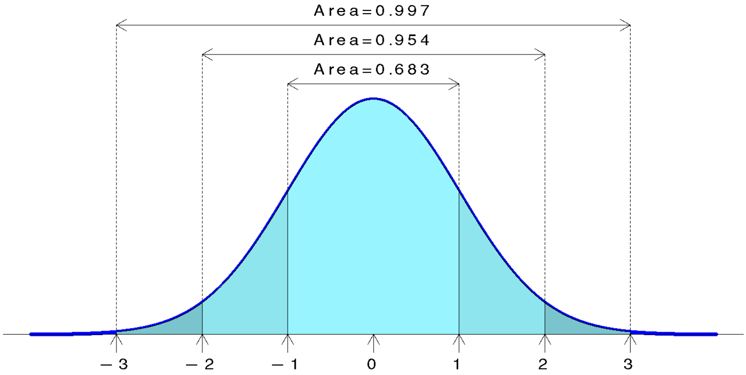
The Normal Distribution



$N(\mu, \sigma)$



$N(0, 1)$: The Standard Normal Distribution



The Standard Normal Tables

The Standard Normal Probabilities can be found in Appendix B, Table 3. The leftmost column indicates the the first two digits of z and the top row indicates the third digit of z. Thus, the entry corresponding to the row -1.0 and the column .02 is $P[Z \leq -1.01]$. Below is the part of the Standard Normal Table.

	The Hundredths Place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5190	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7157	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7969	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8513	.8554	.8577	.8529	.8621

Using the Standard Normal Tables

Example

1. *Find* $P[Z < 1.52]$ *and* $P[Z > 1.75]$.
2. *Find* $P[-1.65 \leq Z \leq 1.65]$.
3. *Find* z *so that* $P[Z > z] \approx 0.10$.
4. *Find* z *so that* $P[-z \leq Z \leq z] \approx 0.05$.

	The Hundredths Place of z									
z	*.00*	*.01*	*.02*	*.03*	*.04*	*.05*	*.06*	*.07*	*.08*	*.09*
1.0	*.8413*	*.8438*	*.8461*	*.8485*	*.8508*	*.8513*	*.8554*	*.8577*	*.8529*	*.8621*
1.1	*.8643*	*.8665*	*.8686*	*.8708*	*.8729*	*.8749*	*.8770*	*.8790*	*.8810*	*.8830*
1.2	*.8849*	*.8869*	*.8888*	*.8907*	*.8925*	*.8944*	*.8962*	*.8980*	*.8997*	*.9015*
1.3	*.9032*	*.9049*	*.9066*	*.9082*	*.9099*	*.9115*	*.9131*	*.9147*	*.9162*	*.9177*
1.4	*.9192*	*.9207*	*.9222*	*.9236*	*.9215*	*.9265*	*.9279*	*.9292*	*.9306*	*.9319*
1.5	*.9332*	*.9345*	*.9357*	*.9370*	*.9382*	*.9394*	*.9406*	*.9418*	*.9492*	*.9441*
1.6	*.9452*	*.9463*	*.9474*	*.9484*	*.9495*	*.9505*	*.9515*	*.9525*	*.9535*	*.9545*
1.7	*.9554*	*.9564*	*.9573*	*.9582*	*.9591*	*.9599*	*.9608*	*.9616*	*.9625*	*.9633*
1.8	*.9641*	*.9649*	*.9656*	*.9664*	*.9671*	*.9678*	*.9686*	*.9693*	*.9699*	*.9706*
1.9	*.9713*	*.9719*	*.9726*	*.9732*	*.9738*	*.9744*	*.9750*	*.9756*	*.9761*	*.9767*
2.0	*.9772*	*.9778*	*.9783*	*.9788*	*.9793*	*.9798*	*.9803*	*.9808*	*.9812*	*.9817*

For Next Time

- Read Section 6.1, 6.2, and 6.3 from Johnson and Bhattacharyya
- Group Problems:

Group	1	2	3	4	5
Problem	6.1	6.3	6.5	6.9	6.11

Group	6	7	8	9	10
Problem	6.13	6.15	6.17	6.19	6.21