STAT 100 Lecture 12: Calculations and Approximations with Normal Distributions

Nate Strawn

http://www.math.umd.edu/∼nstrawn/

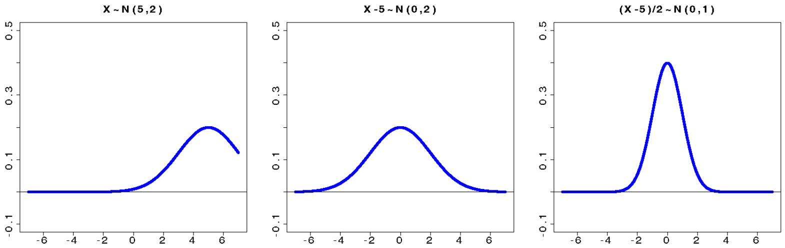
Last Time...

1. Continuous Distributions
   - Probability Density Function
   - Probability Density Curve
2. The Normal Distribution $N(\mu, \sigma)$
   - The Standard Normal $N(0, 1)$
3. Using The Standard Normal Table

Today's Agenda

1. Calculations using Normal Distributions (Section 6.4)
2. The Normal Approximation to the Binomial Distribution (Section 6.5)
3. Examples

Standardizing a Normal Distribution



Standardizing a Normal Distribution

Fact

If X is distributed as $N(\mu, \sigma)$, then the standardized variable

$$Z = \frac{X - \mu}{\sigma}$$

is distributed as $N(0, 1)$.

Fact

If X is distributed as $N(\mu, \sigma)$, then

$$P[a \leq X \leq b] = P\left[\frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma}\right].$$

Here, Z is distributed as $N(0, 1)$.

Example

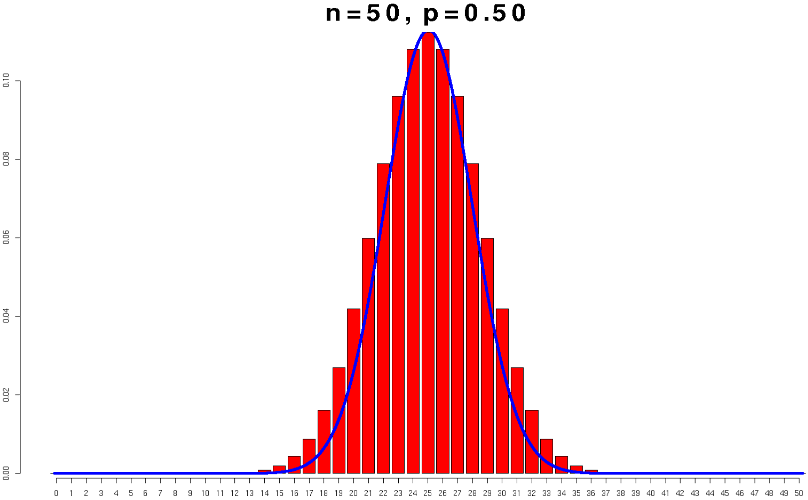
Example

The heights (in inches) of U.S. females age 20+ is normally distributed with $\mu = 63.8$ *and* $\sigma = 0.06$*. The heights of U.S. males age 20+ is normally distributed with* $\mu = 69.3$ *and* $\sigma = 0.05$[a]*.*

1. *What is the probability that a U.S. person over 20 has height between 5 and 6 feet?*
2. *Find the probability that a U.S. female over 20 is taller than 5'5".*
3. *Find the* 95^{th} *percentile for both female and male heights.*

[a] M.A. McDowell, C.D. Fryar, R. Hirsch and C.L. Ogden, Anthropometric reference data for children and adults: US population, 1999-2002, Adv Data 361 (2005)

The Binomial Distribution is Almost Normal...



The Normal Approximation to the Binomial

When np and $n(1-p)$ are both large, the binomial distribution is well approximated by the normal distribution $N(np, \sqrt{np(1-p)})$. That is,

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

is approximately normal with parameters $\mu = 0$ and $\sigma = 1$.

Continuity Correction

Assuming that $X_{binomial}$ has binomial distribution with parameters n and p, and also that X_{normal} has normal distribution with $\mu = np$ and $\sigma = \sqrt{np(1-p)}$, then

$$P[a \leq X_{binomial} \leq b] \approx P[a - 0.5 \leq X_{normal} \leq b + 0.5].$$

Example: Binomial Approximation

Example

Suppose X has binomial distribution with parameters $n = 200$ and $p = 0.3$

1. *Is the normal approximation suitable in this case?*
2. *Estimate $P[X = 60]$.*
3. *Estimate $P[90 \leq X \leq 120]$.*
4. *Find x so that $P[60 - x \leq X \leq 60 + x] = 0.05$.*

For Next Time

- MINITAB Project 03! Due Next Monday (Oct. 27).
- Read Section 6.4, 6.5, and 7.1-7.3 from Johnson and Bhattacharyya
- Group Problems:

Group	1	2	3	4	5
Problem	6.25	6.27	6.29	6.31	6.33
Group	6	7	8	9	10
Problem	6.39	6.41	6.43	6.47	6.49