STAT 100 Lecture 16 : Testing Hypotheses about population mean

Nate Strawn

November 5

Last Time...

1. A confidence interval when normal population and σ known.
2. Interpretation of confidence intervals.
3. Large sample confidence interval for μ.

Today's Agenda

1. Formulating the hypotheses.
2. Test criterion and rejection region.
3. Two types of error and their probabilities.
4. Performing a test.
5. P-value: how strong is a rejection of H_0?
6. Examples.

Formulating the hypothesis.

- The claim or research hypothesis that we wish to establish -**alternative hypothesis** H_1.
- The opposite statement, one that nullifies the research hypothesis-**null hypothesis** H_0.

Definition

When our goal is to establish an assertion with substantive support obtained from the sample, the negation of the assertion is taken to be the **null hypothesis** H_0 *and the assertion itself is taken to be the* **alternative hypothesis** H_1.

Example

1. A company's mixed nuts are sold in cans and the label says that 25% of the contents is cashews. Suspecting that this might be an overstatement, an inspector takes a random sample of 35 cans and measures the percent weight of cashews [i.e., 100(weight of cashews/weight of all nuts)] in each can.

2. Biological oxygen demand (BOD) is an index of pollution that is monitored in the treated effluent of paper mills on a regular basis. From 43 determinations of BOD (in pounds per day) at a particular paper mill during the spring and summer months of 1992, the mean and standard deviation were found to be 3246 and 757, respectively. The company had set the target that the mean BOD should be 3000 pounds per day. Do the sample data indicate that the actual amount of BOD is significantly off the target?

Formulating the hypothesis. Court trail analogy.

Example

	Court Trial
Null hypothesis (H_0):	*Not guilty*
Alternative hypothesis (H_1):	*Guilty*

In this scenario, false rejection of H_0 is a more serious error than failing to reject H_0 when H_1 is true.

A decision rule

A decision rule, or **a test of the null hypothesis,** specifies a course of action by stating what sample information is to be used and how it is to be used in making a decision.

Decision

Either

reject H_0 and conclude that H_1 is substantiated

or

retain H_0 and conclude that H_1 fails to be substantiated

Test criterion and rejection region

Suppose the testing problem is
Test: $H_0 : \mu = 270$ versus $H_1 : \mu < 270$
So, a reasonable decision rule should be of the form:

- Reject H_0 if $\overline{X} \leq c$
- Retain H_0 if $\overline{X} > c$

Definition

This decision rule is conveniently expressed as $R : \overline{X} \leq c$, where R stands for the rejection of H_0 and is called the **rejection region** *or* **critical region,** *and the cutoff point c is called the* **critical value.**

Test criterion and rejection region

A serious error can be when we reject H_0 while $\mu = 270$. For an adequate protection against this kind of error, we must ensure that $P[\overline{X} \leq c]$ is very small when $\mu = 270$. For example, suppose that we wish to hold a low probability of $\alpha = .05$ for a wrong rejection of H_0. Then our task is to find the c that makes

$P[\overline{X} \leq c] = .05$ when $\mu = 270$

Test criterion and rejection region

Suppose $n = 38$ and $\sigma = 24$. Then the distribution of $\overline{X}$ is approximately normal with mean μ and standard deviation $\sigma/\sqrt{n}$ so, the distribution of $\overline{X}$ is $N(270, 24/\sqrt{38})$ and

$$Z = \frac{\overline{X} - 270}{24/\sqrt{38}}$$

has the $N(0,1)$ distribution.

Since $P[Z \leq -1.645] = .05$ the cutoff c on $\overline{x}$ scale must be 1.645 standard deviations below $\mu_0 = 270$. That is

$$c = 270 - 1.645 \times 24/\sqrt{38} = 270 - 6.40 = 263.60$$

Test criterion and rejection region

So, the rejection region is

$$R : \overline{X} \leq 263.6$$

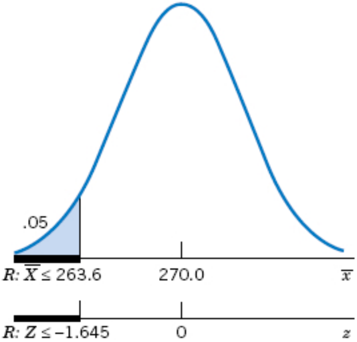

Figure 7
Rejection region with the cutoff $c = 263.6$.

Test criterion and rejection region

Definition

The random variable $\overline{X}$ *whose value serves to determine the action is called the* **test statistic.**
A **test of the null hypothesis** *is a course of action specifying the set of values of a test statistic* $\overline{X}$*, for which* H_0 *is to be rejected.*
This set is called the **rejection region** *of the test.*

Two types of error and their probabilities

	Unknown True	Situation
Decision Based on Sample	H_0 True $\mu = 270$	H_1 True $\mu < 270$
Reject H_0	Wrong rejection of H_0 (Type I error)	Correct decision
Retain H_0	Correct decision	Wrong retention of H_0 (Type II error)

Two types of error and their probabilities

Definition

Type I error: *Rejection of H_0 when H_0 is true.*
Type II error: *Nonrejection of H_0 when H_1 is true.*
$\alpha =$ *Probability of making type I error (also called* **the level of significance***)*
$\beta =$ *Probability of making type II error*

In our problem the rejection region is of the form $R : \overline{X} \leq c$; so that,

$$\alpha = P(\overline{X} \leq c) \text{ when } \mu = 270 \quad H_0 \text{ is true}$$

$$\beta = P(\overline{X} > c) \text{ when } \mu < 270 \quad H_1 \text{ is true}$$

Two types of error and their probabilities

$\alpha =$ Probability of making type I error, $\beta =$ Probability of making type II error.

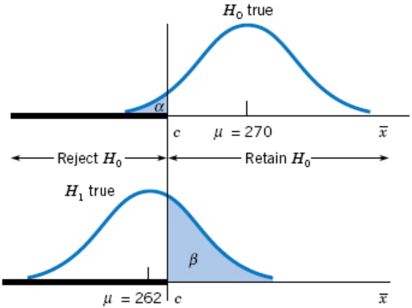

Figure 8 The error probabilities α and β.

The probability β depends on the numerical value of μ that prevails under H_1. Type II error probability β is the shaded area under the normal curve that has $\mu = 262$, a case of H_1 being true.

For Next Time

- Read Section 8.4 from Johnson and Bhattacharyya
- Group Problems:

Group	1	2	3	4	5
Problem	8.37	8.41	8.49	8.51	8.53
Group	6	7	8	9	10
Problem	8.37	8.41	8.49	8.51	8.53