STAT 100 Lecture 21: Hypothesis Tests for μ when X is Normal

Nate Strawn

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Last Time...

1. Student's t-Distribution
   - Using the t-table
   - For large $d.f. = n - 1$, the t-distribution looks like the standard normal
2. Confidence Intervals for μ when X is Normal
   - Can't emphasize **when X is Normal** enough

Student's t Distribution

Definition

If $X_1, \ldots, X_n$ is a random sample from $N(\mu, \sigma)$,

$$\overline{X} = \frac{1}{n}\sum X_i, \text{ and } S^2 = \frac{\sum(X_i - \overline{X})^2}{n-1},$$

then the distribution of

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

is called Student's t-distribution with $n-1$ degrees of freedom, and we write $d.f. = n-1$.

Confidence Intervals for μ when X is Normal

A 100(1 − α)% Confidence Interval for a Normal Population Mean

$$\left(\overline{X} - t_{\alpha/2}\frac{S}{\sqrt{n}}, \quad \overline{X} + t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

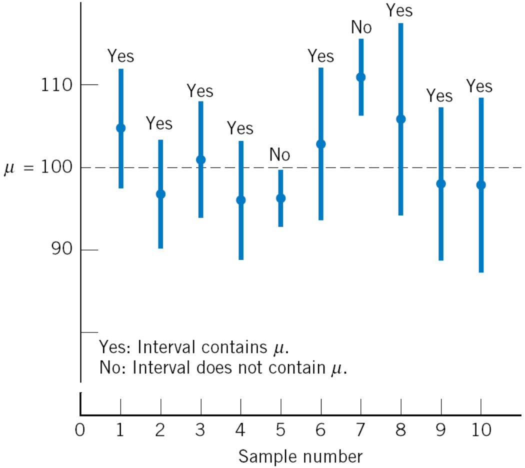
where $t_{\alpha/2}$ is the upper $\alpha/2$ point of the t distribution with d.f. $= n - 1$.

Today's Agenda

1. Interpretation of Confidence Intervals for μ when X is Normal
2. Hypothesis Tests for μ when X is Normal
3. Examples.

Interpretation of Confidence Intervals

A $100(1-\alpha)\%$ confidence interval contains μ with probability $(1-\alpha)$.



Example: Another Confidence Interval

Example

Temperature highs over nine days were 57, 53, 52, 57, 60, 61, 59, 54, and 60 in degrees Fahrenheit. Thus, we have $\overline{x} = 57$ *and* $S = 3.32$.

1. *Find a* 90% *confidence interval for the mean daily high.*
2. *Is* μ *in this interval?*

Hypotheses Tests for μ—Small Samples

To test $H_0: \mu = \mu_0$ concerning the mean of a **normal population,** the test statistic is

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

which has Student's t distribution with $n - 1$ degrees of freedom:

$$\begin{array}{ll} H_1: \mu > \mu_0 & R: T \geq t_\alpha \\ H_1: \mu < \mu_0 & R: T \leq -t_\alpha \\ H_1: \mu \neq \mu_0 & R: |T| \geq t_{\alpha/2} \end{array}$$

The test is called a **Student's *t* test** or simply a ***t* test.**

Example: One-Sided Hypothesis Tests

Example

Temperature highs over nine days were 57, 53, 52, 57, 60, 61, 59, 54, and 60 in degrees Fahrenheit. Thus, we have $\overline{x} = 57$ *and* $S = 3.32$.

1. *Is there significant evidence that the actual mean is over 54 degrees Fahrenheit?*
2. *Is there significant evidence that the actual mean is below 60 degrees Fahrenheit?*

Example: Two-Sided Hypothesis Test

Example

Temperature highs over nine days were 57, 53, 52, 57, 60, 61, 59, 54, and 60 in degrees Fahrenheit. Thus, we have $\overline{x} = 57$ *and* $S = 3.32$.

1. *At the* $\alpha = 0.1$ *level, is there significant evidence that the actual mean is exactly 55 degrees Fahrenheit?*
2. *What is the relationship between the last answer and the confidence interval we calcluated at the beginning of the lecture?*

For Next Time

- Read Section 9.4 from Johnson and Bhattacharyya
- Group Problems:

Group	1	2	3	4	5
Problem	9.11	9.13	9.15	9.17	9.27
Group	6	7	8	9	10
Problem	9.11	9.13	9.15	9.17	9.27