STAT 100 Lecture 22: The Relationship Between Hypothesis Tests and Confidence Intervals

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Last Time...

1. Interpretation of Confidence Intervals for μ when X is Normal

2. Hypothesis Tests for μ when X is Normal

Hypotheses Tests for μ—Small Samples

To test $H_0: \mu = \mu_0$ concerning the mean of a **normal population,** the test statistic is

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

which has Student's t distribution with $n - 1$ degrees of freedom:

$$\begin{aligned} H_1&: \mu > \mu_0 & R&: T \geq t_\alpha \\ H_1&: \mu < \mu_0 & R&: T \leq -t_\alpha \\ H_1&: \mu \neq \mu_0 & R&: |T| \geq t_{\alpha/2} \end{aligned}$$

The test is called a **Student's *t* test** or simply a ***t* test.**

Today's Agenda

1. Relationship between Hypothesis Tests and Confidence Intervals
2. Examples.

Relationship between Hypothesis Tests and Confidence Intervals

We can make two-sided hypotheses concerning the expected value of X:

$$H_0 : \mu = \mu_0 \text{ versus } H_1 : \mu \neq \mu_0.$$

The rejection region for a $100(1-\alpha)\%$ hypothesis test is then

$$\overline{x} \leq \mu_0 - t_{\alpha/2}\frac{S}{\sqrt{n}} \text{ or } \mu_0 + t_{\alpha/2}\frac{S}{\sqrt{n}} \leq \overline{x},$$

so the "acceptance" region is

$$\mu_0 - t_{\alpha/2}\frac{S}{\sqrt{n}} < \overline{x} < \mu_0 + t_{\alpha/2}\frac{S}{\sqrt{n}}.$$

Relationship between Hypothesis Tests and Confidence Intervals

Note that the bounds of this inequality are exactly the bounds of a $100(1-\alpha)\%$ confidence interval for μ:

$$\overline{x} - t_{\alpha/2}\frac{S}{\sqrt{n}} < \mu_0 < \overline{x} + t_{\alpha/2}\frac{S}{\sqrt{n}}.$$

We can now interpret the decision rule of the hypothesis test in terms of confidence intervals: Retain H_0 only when the $100(1-\alpha)\%$ confidence interval contains μ_0.

This means that we get a hypothesis test for μ for free when we compute the confidence interval!

Example: Relation between a 99% Confidence Interval and a Two-Sided $\alpha = 0.005$ test

Example

Consider a random sample of size $n = 27$ from a normal population. Suppose $\overline{x} = 12$ and $S = 1.5$.

1. *Compute a 99% confidence interval for μ.*
2. *Test the hypothesis $H_0 : \mu = 15$ versus $H_1 : \mu \neq 15$.*

For Next Time

- Read Sections 10.1 and 10.2 from Johnson and Bhattacharyya

- Group Problems:

Group	1	2	3	4	5
Problem	9.33	9.35	9.37	9.57	9.67
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