Homework #6: Due on 11/26 (A proper subset of the problems will be selected for grading.)

A) Solve the following problems from the textbook. 4.2, 4.4 a–c; 4.12, 4.13, 4.19, 4.20, 4.21, 4.36, 4.37,

B) Let $f$ be a function of bounded variation on $[0, 1]$ such that $x \rightarrow V(f, [0, x])$ is absolutely continuous. Prove that $f$ is absolutely continuous on $[0, 1]$.

C) For a function $f : [0, 1] \rightarrow \mathbb{R}$, prove that $f$ is Lipschitz if and only if it is absolutely continuous and there is an $M \geq 0$ for which

$$|f'(x)| \leq M \quad m - a.e. \quad x \in [0, 1].$$

D) (This will not be graded).

D-1) Let $f : [a, b] \rightarrow \mathbb{R}$ be a real-valued Lebesgue measurable function. If $E$ is a measurable subset of $[a, b]$ and $f$ is differentiable at every point of $E$, prove that $m^*(f(E)) \leq \int_E |f'| dm$.

D-2) Prove that if $f : [a, b] \rightarrow \mathbb{R}$ is differentiable everywhere on $[a, b]$ and $f' \in L^1_m([a, b])$, then $f \in AC([a, b])$. 