Solve the following problems from the textbook. A proper subset of the problems will be selected for grading.

1. 5.37 (a, d); 5.39; 5.42.

2. Let $X = \mathbb{R}$, and $\mathcal{A} = \mathcal{M}$ be the set of Lebesgue measurable subsets of $\mathbb{R}$. Consider, the functions $f(x) = e^x, x \in \mathbb{R}, g(x) = x^3, x \in \mathbb{R}$, and

$$h(x) = \begin{cases} 
0 & : x \leq 0 \\
x & : 0 \leq x \leq 1 \\
1 & : 1 \leq x. 
\end{cases}$$

Recall that for any right-continuous function $k$, $\mu_k$ is the measure defined on $\mathcal{M}$ such that $\mu_k((a, b]) = k(b) - k(a)$.

a. Give a Lebesgue decomposition of $\mu_f$ with respect to $\mu_h$.

b. Show that $\mu_g \ll \mu_f$, and find its R-N derivative $\frac{d\mu_g}{d\mu_f}$.