Math 631, Spring 2015
Homework #6: Due on 3/10

Solve the following problems from the textbook. A proper subset of the problems will be selected for grading.

1. 5.36, 5.37, 5.39, 5.52

2. Let $\mu, \nu$ be two measures on a measurable space $(X, \mathcal{A})$, with $\nu \ll \mu$, and let $\lambda = \mu + \nu$. If $f = \frac{d\nu}{d\lambda}$, prove that $0 \leq f < 1 \text{a.e.}$ and $\frac{d\nu}{d\mu} = \frac{1}{1-f}$.

3. Let $X = \mathbb{R}$, and $\mathcal{A} = \mathcal{M}$ be the set of Lebesgue measurable subsets of $\mathbb{R}$. Consider, the functions $f(x) = e^x, x \in \mathbb{R}$, $g(x) = x^3, x \in \mathbb{R}$, and

   $$h(x) = \begin{cases} 
   0 & : x \leq 0 \\
   x & : 0 \leq x \leq 1 \\
   1 & : 1 \leq x.
   \end{cases}$$

Recall that for any right-continuous function $k$, $\mu_k$ is the measure defined on $\mathcal{M}$ such that $\mu_k((a,b]) = k(b) - k(a)$.

a. Give a Lebesgue decomposition of $\mu_f$ with respect to $\mu_h$.

b. Show that $\mu_g \ll \mu_f$, and find its R-N derivative $\frac{d\mu_g}{d\mu_f}$.