MATH 240 Homework
Bringing the Pain # 1

Instructions: Do as much as you can by Tuesday, 2005 2/15. Prepare your answers on a separate sheet of paper. These problems should be useful for your exam.

The basics (wax on... wax off...)

1. Give the definitions for the following:

a. A *linear combination* of $v_1, \ldots, v_k$ is ________.

b. $\text{Span}(v_1, \ldots, v_k)$ is ________.

c. The set $\{v_1, \ldots, v_p\}$ is *linearly independent* if ________.

d. Consider an $m \times n$ matrix A. If $A =$____ and $x =$____, then $Ax =$ ________ (in terms of the columns of A and the entries of x).

e. A transformation $T : U \to V$ is *linear* if ________.

f. A mapping $T : U \to V$ is *one-to-one* if ________.

g. A mapping $T : U \to V$ is *onto* if ________.

2. Explain as clearly as you can what the terms in #1 mean. Illustrate by example or by diagram if it helps.

Important theorems below!

3. Fill in the blanks.

Let us consider an $m \times n$ matrix A. The following statements are equivalent:

a. $\mathbb{R}^- = \text{span(columns of } A) = \text{Col } A = \{Ax : x \text{ is in } \mathbb{R}^-\}$.

b. ____ vector in $\mathbb{R}^-$ is a linear combination of ________.

c. The matrix equation $Ax = b$ ________ for ____ b in $\mathbb{R}^-$.

d. A has ___ pivots.

e. The linear transformation $T : \mathbb{R}^- \to R^-$, defined by $T(x) = Ax$, is ____.

Note that if any of the statements a-e hold, the number of rows of A is ____ than the number of columns of A.

4. Fill in the blanks.

For an $m \times n$ matrix A, the following are equivalent:

a. The columns of A form a linearly independent set.

b. $Ax = 0$ implies ________.

c. The solution set for $Ax = b$ has either ___ element(s) or ___ element(s).

d. A has ___ pivots.

e. The linear transformation $T : \mathbb{R}^- \to R^-$, defined by $T(x) = Ax$, is ____.

Note that if any of the statements a-e hold, m ___ n.

The statements in 3 and 4 tell us that counting pivots is very useful. Many questions about a matrix can be answered just by row reducing it to an echelon form to see its pivots. Play with a few matrices and see which of the above properties must hold in each case. Try "skinny, fat, and square" matrices.

Ho hum. (1.5)
5. What is the parametric vector expression for the line passing through x parallel to y? How about for the line passing through u and v?

6. Explain as clearly as you can the relation between the solution set for $Ax = 0$ and the solution set for $Ax = b$.

7. Behold the might of linear transformations! (1.8 and 1.9)
a. Let the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by:

$$T(e_1) = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \quad \text{and} \quad T(e_2) = \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}.$$

What does $T\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ equal to?
What is the standard matrix of the transformation T (see p. 83)?
(Recall that $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.)

b. Let the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ satisfy:

$$T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad T\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}.$$

What are $T(e_1)$ and $T(e_2)$ equal to? Write down the standard matrix of T.
Use the standard matrix to calculate $T\begin{pmatrix} -5 \\ 3 \end{pmatrix}$.