HOMEWORK 2
NOTHING GOOD

Instructions: This covers sections 2.1-2.3, 2.5, 3.1-3.2, and parts of Chapter 4. The material in the last sections of Chapter 2, from 2.7 onwards, also appear in Chapter 4. Refer to Chapter 4 for this material in the assignment below. Answer all questions, fill in all blanks, and follow the advice. Failure to comply will result in Nothing Good. Submit your answers on the Tuesday before the next exam, 2005 March 8.

1. Matrix multiplication:
a. (definition) Let A be an $m \times n$ matrix and B an $n \times p$ matrix, with columns given by $B =$ ______. Then the columns of AB are given by $AB =$ ______.
Furthermore, because of the way we defined matrix-vector multiplication, each column of AB is a(n) ____________.
b. Write a similar statement as in part (a) about the rows of AB.
c. How can we express the entry in ith row and jth column of AB?
d. What is the column-row expansion of AB?

2. inverse of A:
a. (definition) A matrix A of size __×__ is *invertible* if ________.
b. Invertible matrix theorem (IMT)
Examine #3 and #4 in **Homework 1** and compare them with the statements of the IMT in the book. Each of the statements in the IMT (except one or two) can be listed as a statement for #3 or for #4 if we take the statement to apply to an $m \times n$ matrix instead. Write the statements of the IMT not already listed in #3 or #4 and indicate whether they belong with #3 or #4 or neither. You will need to modify some of those statements for clarity so that they apply to an $m \times n$ matrix A.
c. transpose of A:
(definition) If the columns of an $m \times n$ matrix A are given by $A = [\mathbf{a}_1 \dots \mathbf{a}_n]$, then the *transpose* of A, A^T, is an __×__ matrix whose rows are given by $A =$ ________.
d. properties of the transpose and inverse:
Copy these down and write what they equal to (according to a theorem):

$$(A^T)^T, (A+B)^T, (cA)^T, (AB)^T, (A^{-1})^{-1}, (AB)^{-1}, (A^T)^{-1}$$

3. Subspaces of a vector space
a. (definition) Let W be a ____ of a vector space V. Then W is a *subspace* of V if ________.
b. Is the span of a collection of vectors a subspace?

4. The null space of a matrix and the kernel of a (linear) transformation
a. (definition) The *null space* of a matrix A is ________.
(definition) The *kernel* of a transformation $T : U \to V$ is ________.
b. The null space of an $m \times n$ matrix is a subspace of $\mathbb{R}^{-}$.
The kernel of a linear transformation $T : U \to V$ is a subspace of (the vector space) ____.
c. Write a statement about the null space of an $m \times n$ matrix A that is equivalent to either

#3 or #4 of **Homework 1** and indicate where it belongs (#3 or #4).
Do the same for the kernel of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, defined by $T(x) = Ax$.

5. The column space of a matrix and the range of a (linear) transformation
a. (definition) The *column space* of an $m \times n$ matrix A is ________.
(definition) The *range* of a transformation $T : U \to V$ is ________.
b. The column space of an $m \times n$ matrix is a subspace of $\mathbb{R}^-$.
The range of a linear transformation $T : U \to V$ is a subspace of (the vector space) ____.
c. Write a statement about the column space of an $m \times n$ matrix A that is equivalent to either #3 or #4 of **Homework 1** and indicate where it belongs (#3 or #4).
Do the same for the range of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, defined by $T(x) = Ax$.

6. basis and dimension (of a subspace)
a. (definition) Let W be a subspace of V. A *basis* for W is ________.
(definition) The *dimension* of a vector space (or subspace) is ________.
b. Write a statement involving 'basis', equivalent to the IMT.
c. State the Basis Theorem.

7. The rank and nullity of a matrix
a. (definition) The *rank* of a matrix is ________.
(definition) The *nullity* of a matrix is ________.
b. Write a statement about the rank of an $m \times n$ matrix A equivalent to #3 or #4 of **Homework 1** and indicate where it belongs. Ditto for nullity.
c. The rank of an $m \times n$ matrix is also the number of ________ in the matrix and does not exceed ____.
d. The nullity of an $m \times n$ matrix is the number of ________ in the matrix and does not exceed ____.
e. State the Rank Theorem.

8. The coordinates of a vector relative to a basis
a. (definition) Let $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ be a basis for the vector space V. The *coordinates* of a vector $\mathbf{v} \in V$ are the weights ________ such that $\mathbf{v} =$ ________.
The *coordinate vector* of $\mathbf{v}$ is $[\mathbf{v}]_{\mathcal{B}} =$ ________.
b. While $\mathbf{v}$ is a vector in the vector space V of dimension ____, $[\mathbf{v}]_{\mathcal{B}}$ is a vector in $\mathbb{R}^-$. In fact, the coordinate mapping $[\ \cdot\ ] : V \to \mathbb{R}^-$, $\mathbf{v} \mapsto [\mathbf{v}]_{\mathcal{B}}$ is an isomorphism.
c. Computing the coordinates of a vector in $\mathbb{R}^n$ relative to the standard basis is easy. Make sure you see why.
d. Learn how to compute the coordinates of a vector in $\mathbb{R}^n$ relative to any basis (for $\mathbb{R}^n$).

9. LU factorization
Know what it is and how to compute it for an $m \times n$ matrix A.

10. determinants (of square matrices)
Know how to compute determinants through the row reduction process.
Know how to compute determinants by applying properties of the determinant.
a. A(n) ____×____ matrix A is invertible if and only if <u>(something about det A)</u>.
b. Copy these down and write what they equal. Make sure your answer applies to square

matrices of any size. c is a scalar and I is an identity matrix.

$$\det(AB), \det(cA), \det I, \det(-I)$$

11. MATLAB
a. In MATLAB, how do we check if a matrix is invertible?
b. If we need to compute the inverse of a matrix, what command do we use?
c. If we want to solve the equation Ax = b for an invertible matrix, what command do we use?
d. What is the command to compute the LU factorization of a matrix A?

12. Other Things
a. What is the formula for calculating the inverse of a 2×2 matrix?
b. Learn how to compute the inverse of a matrix by hand through row reduction.
c. Know how to compute AB by hand.
d. Know how to compute bases for Col A and for Nul A.