AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #1 Due June 12, 2009

1. Ex.1.35, p.46, *Numerical Computing with MATLAB* . Modify the programs by inserting a counter that will count the lines of output. Do not hand in any output. Just answer the questions.
2. Ex.1.38, p.46, *Numerical Computing with MATLAB*
3. Ex.1.39, p.47, *Numerical Computing with MATLAB* Explain, using a fundamental property of the sine function, how one could modify **powersin** so it would compute $\sin x$ accurately for <u>all</u> real x.
4. Suppose a computer carries three decimal digits and rounds. If x and y are machine numbers, define the machine version of addition $x \oplus y$ to be the result of adding x and y and rounding to three digits. For example

   $$49.3 \oplus 57.4 = 107.$$

   Define machine multiplication $x \otimes y$ similarly. For example,

   $$1.23 \otimes 4.86 = 5.98.$$

   Constuct examples to show that, in general, the following statements are <u>not</u> true:
   (a) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$,
   (b) $(x \otimes y) \otimes z = x \otimes (y \otimes z)$,
   (c) $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$.
5. For a set of measurements $x_1, x_2, \ldots, x_N$, the sample mean $\bar{x}$ is defined to be

   $$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

   The sample standard deviation s is defined to be

   $$(N-1)s^2 = \sum_{i=1}^{N}(x_i - \bar{x})^2.$$

   Another expression,

   $$(N-1)s^2 = \sum_{i=1}^{N} x_i^2 - \frac{1}{N}\Big(\sum_{i=1}^{N} x_i\Big)^2,$$

   is ofter recommended for hand computation of s. Show that these two expressions for s are mathematically equivalent. Explain why one of them may provide better numerical results than the other, and construct and example to illustrate your point.
6. Using four digit arithmetic, add the following numbers, first in ascending order (from smallest to largest) and then in descending order. In doing so round off the partial

sums to four significant figures. Compare your results with the correct sum $x = 0.113731041e+5$. (e+n means 10^n.)

0.1673e+0	0.3875e+1	0.8053e+3
0.2548e+0	0.5478e+2	0.8886e+3
0.2997e+1	0.7113e+2	0.9546e+4

7. Let

$$f(x) = \frac{\ln(1+x) - \sin x - \cos x + 1}{x^3}.$$

Use MATLAB to compute $f(x)$ for $x = 10^{-m}$, $m = 1, 2, \cdots, 12$. According to the theory what is $\lim_{x \to 0} f(x)$? For x near zero what is a better way to compute $f(x)$? (Hint: Use Taylor's theorem on the numerator.)

8. Write a program to compute the first 60 terms in the sequence given by the difference equation

$$x_{k+1} = 2.25x_k - 0.5x_{k-1} \tag{1}$$

with the starting values

$$x_1 = \frac{1}{3} \text{ and } x_2 = \frac{1}{12}.$$

Make a plot of $\log_2 x_k$ (log2 in MATLAB) as a function of k. The exact solution of (1) is given by

$$x_k = \frac{4^{1-k}}{3}$$

which decreases monotonically as k increases. Does your graph confirm this theoretically expected behavior? Can you explain the results? (**Hint:** Find the general solution of (1). To do this, look for solutions of the form $x_k = \alpha^k$ and observe that (1) is linear.)

9. For $\alpha = 0.8717$ and $\beta = 0.8719$ calculate the midpoint m of the interval $[\alpha, \beta]$ by using the formula $m = (\alpha + \beta)/2$. First use four-digit decimal chopped arithmetic, then four digit rounded arithmetic. How reasonable are the answers ? Find another formula for the midpoint and use four-digit decimal (rounded or chopped) arithmetic to calculate the midpoint of $[0.8717, 0.8719]$. Is your formula better or worse ?

10. What is the nearest floating point number to 64 on a base-2 computer with 5-bit mantissas? Show work.