AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #4 Due July 10, 2009

1. Write a MATLAB program to evaluate $I = \int_a^b f(x)\,dx$ using the trapezoidal rule with n subdivisions, calling the result I_n. Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \ldots, 512$.

$$\text{(a)} \quad \int_0^1 \sqrt{16 + x^2}\,dx \qquad \text{(b)} \quad \int_0^1 x^{3/8}\,dx$$

The exact value of the integral in (a) is 4.04128450518694.
Analyze emperically the rate of convergence of I_n to I by calculating the ratios

$$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}} \text{ and } p_n = \frac{\log(R_n)}{\log(2)}$$

In part (b) compute the extrapolated approximation to I,

$$I \doteqdot I_{4n} - \frac{(I_{4n} - I_{2n})^2}{(I_{4n} - I_{2n}) - (I_{2n} - I_n)}$$

for $n = 128$.

2. Repeat problem 1 using Simpson's rule.

3. Apply the corrected trapezoidal rule to the integral in problem 1(a). Compare the results with those of problem 2 for Simpson's rule.

4. Use Gauss-Legendre integration with $n = 2, 4, 8$ nodes to the integrals of problem 1. Compare the results with those for the trapezoidal and Simpson methods.

5. Find approximate values of the integral in problem 1(a) by computing the Romberg integral $I_{32}^{(5)}$ where $I_n^{(0)}$ is the n-panel trapezoid approximation and

$$I_n^{(k)} = \frac{4^k I_n^{(k-1)} - I_{n/2}^{(k-1)}}{4^k - 1}$$

   for n divisible by 2^k.

6. Use the MATLAB function QUADL to find approximate values of the integrals 1(a) and 1(b).

7. Ex. 6.13 p.181 *Numerical Computing with MATLAB.*

8. The 11 point Newton-Cotes integration rule on $[0, 1]$ is

$$\int_0^1 f(x)\,dx \approx \sum_{i=0}^{10} w_i f(\frac{i}{10})$$

with the w_i determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \ldots x^{10}$.

(a) Use MATLAB to find the weights w_i.

(b) Apply the rule to the integrals in 1(a) and 1(b). Note the errors.

9. We wish to estimate the value of

$$I = \int_0^\infty e^{-x} \cos^2 x \, dx = .6$$

(a) Truncate the integral and use QUAD on the finite part.

(b) Try the transformation $x = -\ln t$ on this integral and use QUADL on the new integral. (QUADL will complain but will do it).

(c) Use the 2, 4 and 8 point Gauss-Laguerre rules to estimate the integral. compare your results with parts (a) and (b) above.

10. In a standard shell and tube heat exchanger hot vapor condenses on the tube, maintaining a constant temperature T_s. If the input is at temperature T_1 and the output must be at temperature T_2, then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_\rho dT}{h(T_s - T)}.$$

(All quantities must be in consistent units.) Here T is the temperature in °F.
$T_1 = 0$°F is the inlet temperature.
$T_2 = 180$°F is the desired outlet temperature.
$T_s = 250$°F is the condensate temperature.
m is the fluid flow rate = 45,000 lb/hr.
D is the diameter of the tube = 1.032 in.
c_ρ is the specific heat of the fluid = $(0.53 + 0.00065T)$ BTU/(lb°F).
h is the local heat transfer coefficient $= \frac{0.023k}{D}(\frac{4m}{\pi D \mu})^{0.8}(\frac{\mu c_\rho}{k})^{0.4}$.
k is the thermal conductivity of the fluid = 0.153 BTU/(hr ft°F).
μ is the viscosity of the fluid and has units lb/(ft hr). μ varies with temperature so that

T	0	50	100	150	200
μ	242	82.1	30.5	12.6	5.57

Use spline interpolation to define μ for other values of T and calculate the required length of the heat exchanger.
You will need to use the MATLAB functions SPLINE and QUADL. The answer is about 158.7 ft.