AMSC/CMSC 460 SUMMER 2009

SAMPLE MIDTERM EXAM

1.

(a) In three digit (base 10) arithmetic with rounding, what is the value of machine epsilon?

Use three digit arithmetic with rounding to do the following computations. In each case, find the absolute and relative errors in your results compared to the exact answer.

(b) $(221 - 0.328) - 219$

(c) $(221 - 219) - 0.327$

2. Let

$$A = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ 1 & 1 \end{pmatrix}$$

(a) Find a permutation matrix P, a lower triangular matrix L and an upper triangular matrix U such that $A = PLU$ with the factorization corresponding to Gauss elimination with partial pivoting.

(b) Use the factorization to solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = (\frac{1}{2}, 1)^T$.

3. A quadratic polynomial $p_2(x)$ interpolates data points $(1, 0), (2, y), (3, 14)$. Find y if the coefficient of x^2 in $p_2(x)$ is 3.

4. A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} 2 + 2x - x^3, & 0 \le x \le 1, \\ 3 + b(x-1) + c(x-1)^2 + d(x-1)^3, & 1 \le x \le 2. \end{cases}$$

Find b, c and d.

5. Find the best least squares fit by a linear function $y = \beta_0 + \beta_1 x$ to the data points $(0, 6), (1, 4), (2, 1), (3, 0)$. Make a table showing the observed values of y, the predicted values of y and the errors (the difference between the predicted and observed values). If you did your work correctly, the errors should sum to zero.

6. Consider the integration rule

$$I = \int_0^1 f(x)\, dx \approx A_1 f(1/4) + A_2 f(3/4) = Q_1$$

(a) Show that if we try to determine A_1 and A_2 in such a way that the rule is exact for all first degree polynomials, we are led to the linear system of problem 2.

(b) What result does the rule give when $f(x) = e^x$? Compare the result with the exact value and that obtained by using the (simple) Trapezoid rule, T_1. Which rule does better ?

(c) Is the above rule exact for quadratics ?

(d) By scaling or otherwise find the corresponding rule for an integral over an interval of length h.

$$I = \int_0^h f(x)\, dx \approx \alpha_1 f(h/4) + \alpha_2 f(3h/4) = Q_h$$

(e) Given that the error of the rule is of the form

$$e = I - Q_h = c f''(\zeta) h^3$$

for smooth f, where $0 < \zeta < h$, find the constant c.