

**DUE Tuesday, October 31, 2006**

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Directions:

- Please read all questions carefully.
  - Answer all parts of each question.
  - Please circle or box your final answers.
  - Partial credit is always given for correct methods, partial correct calculations, and correct justification (rules, theorems, definitions, etc).
  - NO CREDIT will be given unless there is adequate work shown or a short explanation is provided.
  - Point values for each question are indicated in parentheses.
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1) Follow the steps below in order to find all zeros of  $f(x)$  and to sketch a graph of the polynomial.

$$f(x) = 3x^3 - 10x^2 + 9x - 2$$

- (1) What is the degree of  $f(x)$ ? \_\_\_\_\_
- (1) What is the leading coefficient of  $f(x)$ ? \_\_\_\_\_
- (2) Use the Leading Coefficient Test to determine the right-hand and left-hand behavior of the graph of  $f(x)$ .
  
- (1) What is the constant term of  $f(x)$ ? \_\_\_\_\_
- (4) List all possible rational zeros of  $f(x)$ .
  
- (1) How many (possible) positive real zeros does  $f(x)$  have? Justify your answer.
  
- (1) How many (possible) negative real zeros does  $f(x)$  have? Justify your answer.
  
- (3) Find all zeros of  $f(x)$ .

i. (1) What is  $f(0)$ ? \_\_\_\_\_

j. (2) Plot all the intercepts and the points corresponding to the following functional values and sketch the graph of  $f(x)$  utilizing all the information you obtained.

$f(-1) = -24$	$f(0.6) = 0.448$	$f(1.5) = -0.875$	$f(3) = 16$
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A large grid of graph paper for plotting the function.				

k. (1) Write the linear factorization form of  $f(x)$  using the zeros you found for the function.

2) Consider the following quadratic equation to answer the questions below.

$$y = -3x^2 - 18x - 15$$

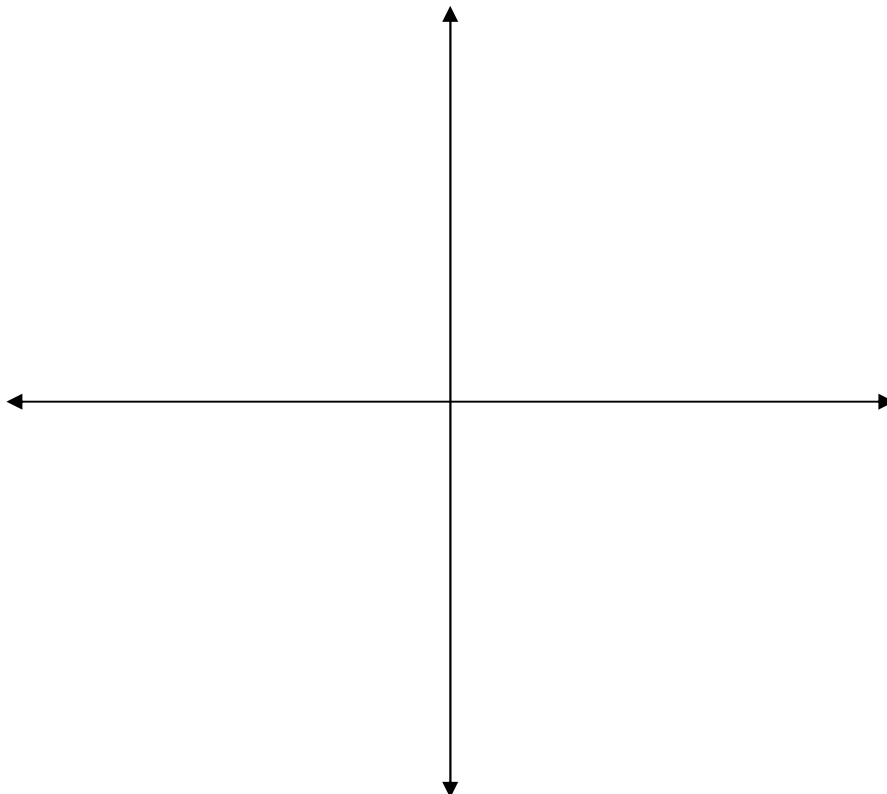
a. (3) Write the equation of the above parabola in standard form.

Recall that standard form is  $y = a(x - h)^2 + k$ .

b. (2) What is the vertex of this parabola?

c. (2) Find the x-intercepts of this quadratic equation.

d. (3) Sketch its graph on the coordinate plane. Sketch means you have the correct vertex, general shape, and THREE points on the graph.



3) a. (3) Use any division method to determine whether or not  $(x-1)$  is a factor of  $g(x) = x^3 + 1$ .

b. (2) Also, what is the value of  $g(1)$ ?

4) Give your answers to the following complex arithmetic operations in standard form,  $a + bi$ .

a. (4)  $(4 - 2i) - [(-3 + 3i) + (1 + i)]$

b. (4)  $(1 - 2i) \div (2 + 3i)$

- 5) (4) Give the equation of any polynomial that has  $\sqrt{2}$ ,  $-1$ , and  $(2-i)$  among its zeros. You do not need to write the polynomial in general form.

- 6) (5) Find the two positive numbers whose product is a maximum if the first number plus three times the second number add up to 54.

Let  $x$  = the first number and  $y$  = the second number. Then  $x + 3y = 54$ . So,  $y = \frac{54-x}{3}$

The product of the two numbers can therefore be computed by:

$$p(x) = (x)\left(\frac{54-x}{3}\right)$$
$$p(x) = 18x - \frac{1}{3}x^2$$

Hint: Notice  $p(x)$  is a quadratic equation and we are looking for the maximum product.

7) Consider the following rational function:  $f(x) = \frac{-3}{2-x}$

a. (4) **Find all asymptotes.**

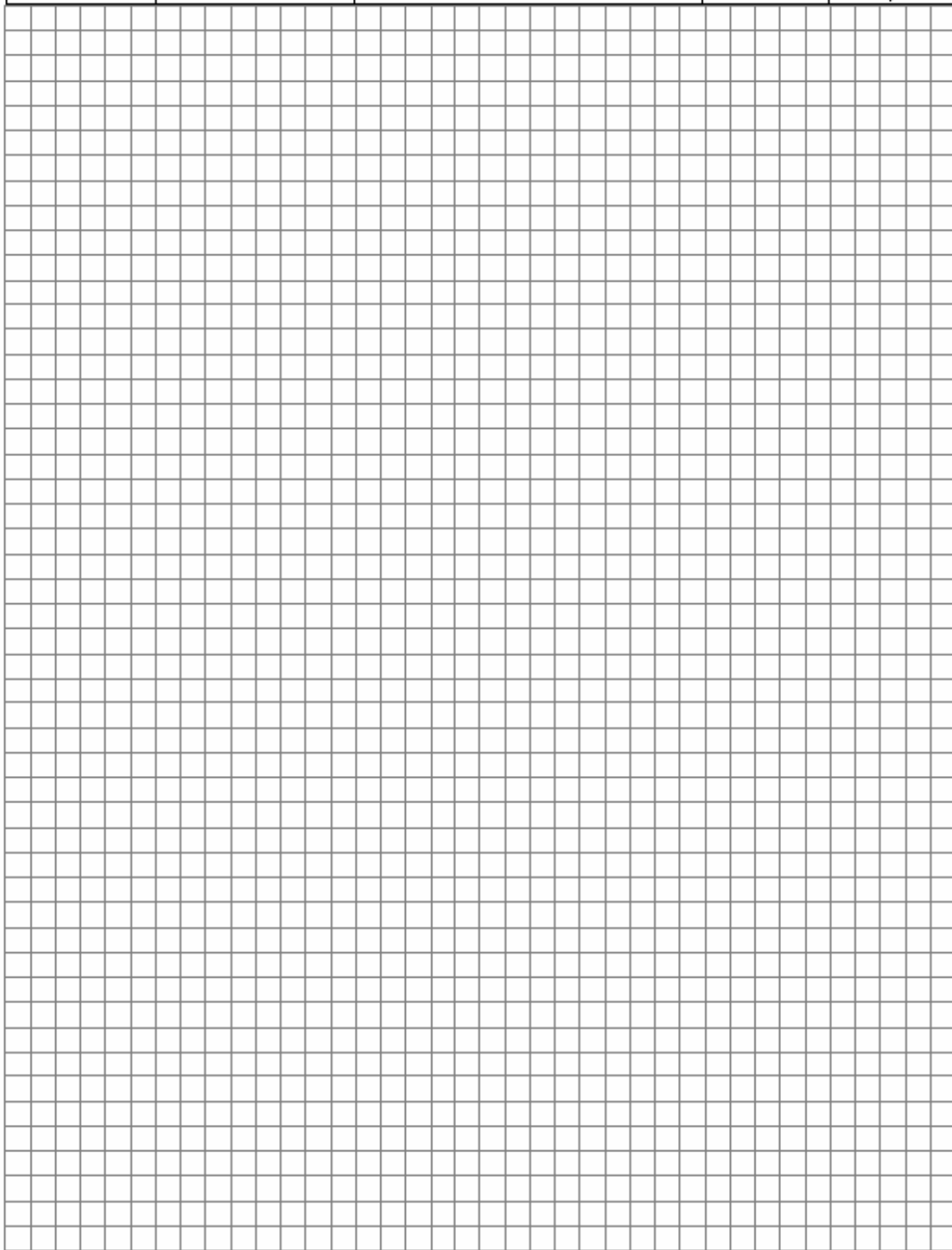
Specify whether they are vertical, horizontal, and/or slant and give their equations.

b. (1) **Find all intercepts.**

c. (3) Given the following points for  $f(x)$ , use these values and the information you have found in parts a and b to complete the graph of  $f(x)$ .

<b>x</b>	-10	-5	1	1.7	2.1	3	5	10	17
<b>y</b>	-0.25	-0.42	-3	-10	30	3	1	0.375	0.2

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- 8) (42/each blank = 3pts) Fill in the blanks with an appropriate word, phrase, or mathematical expression.
- a. If a polynomial  $f(x)$  is divided by  $(x - k)$ , then by the Remainder Theorem, the remainder “r” is equal to \_\_\_\_\_.
- b. If a quadratic polynomial has at least one complex zero, then this function will have \_\_\_\_\_ (how many?) x-intercepts.
- c. The degree of a function that has exactly two real zeros and two complex zeros is \_\_\_\_\_.
- d. **True or False, justify your answer with a short explanation:**
1. A polynomial can have both  $(3 + i)$  and  $(1 - 7i)$  as zeros.
  2. There is **no** complex number that is equal to its own complex conjugate.
  3. The graph of a rational function can **never** cross one its asymptotes.  
(If you answer TRUE, then state why; If FALSE, then provide an example function)
  4. The graph of a ninth degree polynomial can have **no** x-intercepts.
- e. Suppose that  $(2x - 3)$  is a factor of the polynomial  $g(x)$ . Then:
1. \_\_\_\_\_ is a zero of  $g(x)$ .
  2.  $g\left(\frac{3}{2}\right) =$  \_\_\_\_\_.
  3. The point \_\_\_\_\_ is an x-intercept of the graph of  $g(x)$ .
- f. A **function**,  $f$ , from an input set called the \_\_\_\_\_ to an output set called the \_\_\_\_\_, assigns each input element to \_\_\_\_\_ (how many?) element in the output set.
- g.  $i^9 =$  \_\_\_\_\_. (Give the answer as a complex number in standard form.)

BONUS: (+5)

The rational expression  $\frac{-x}{x^2 + 3x + 2}$  has a Partial Fraction Decomposition, meaning it can be equivalently written as

$\frac{A}{x+2} + \frac{B}{x+1}$ , where A and B are two real numbers. Follow the steps in section 2.7 of the text to find the numbers A and B.