

DUE Tuesday, November 28, 2006

* By signing above, I certify that the work done to complete this exam is solely my own, using only the textbook and my notes for reference. If it is determined otherwise, I will accept a grade of zero for this exam.

Directions:

- Please read all questions carefully. Answer all parts of each question. Circle or box your final answers. Partial credit is always given for correct methods, partial correct calculations, and correct justification (rules, theorems, definitions, etc). NO CREDIT will be given unless there is adequate work shown or a short explanation is provided. Point values for each question are indicated in parentheses.
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1) Suppose that a 200kg sample of radioactive material has a half-life of 1650 years, and obeys the exponential decay model. Recall half-life is the time in which half of the original material will decay. Also recall that the amount of material present at any time $t > 0$, follows the equation $P(t) = P_0 e^{kt}$, where k is the decay constant and P_0 is the initial amount of material.

a. (2pts) Determine the value of the decay constant, k , for this particular radioactive material.

b. (4pts) What percentage of the material will remain after 2000 years?

c. (4pts) After how many years will 20kg of material remain out of the original 200kg sample?

- 2) (2pts each) Evaluate the following expressions, writing the solution as an equivalent integer. Note: A calculator is not necessary.

a) $4 \ln e^{0.25} = \underline{\hspace{2cm}}$

b) $2 \log_3 81 = \underline{\hspace{2cm}}$

c) $4 \ln(e^3) - 2 \log_4(4^{16}) - 2 \log_{10} 1000 = \underline{\hspace{2cm}}$

d) $(\ln e)^6 = \underline{\hspace{2cm}}$

e) $2 \ln(e^6) + \ln(e^{-13}) = \underline{\hspace{2cm}}$

f) $2 \ln e^2 - 7 \log_9 1 - 8 \log_{14} 14 + \log_2 16 = \underline{\hspace{2cm}}$

- 3) a. (4pts) What is the x-intercept of the graph of $y = \log_4 x$?

- b. (4pts) Identify the asymptote in the graph of $y = \pi^x$. State the equation and type of asymptote.

4) (3pts each) Find **ALL** exact solutions for x. **Do not approximate** your answers.

a) $\log_x 1 = 0$ x = _____

b) $\left(\frac{2}{3}\right)^x = \frac{9}{4}$ x = _____

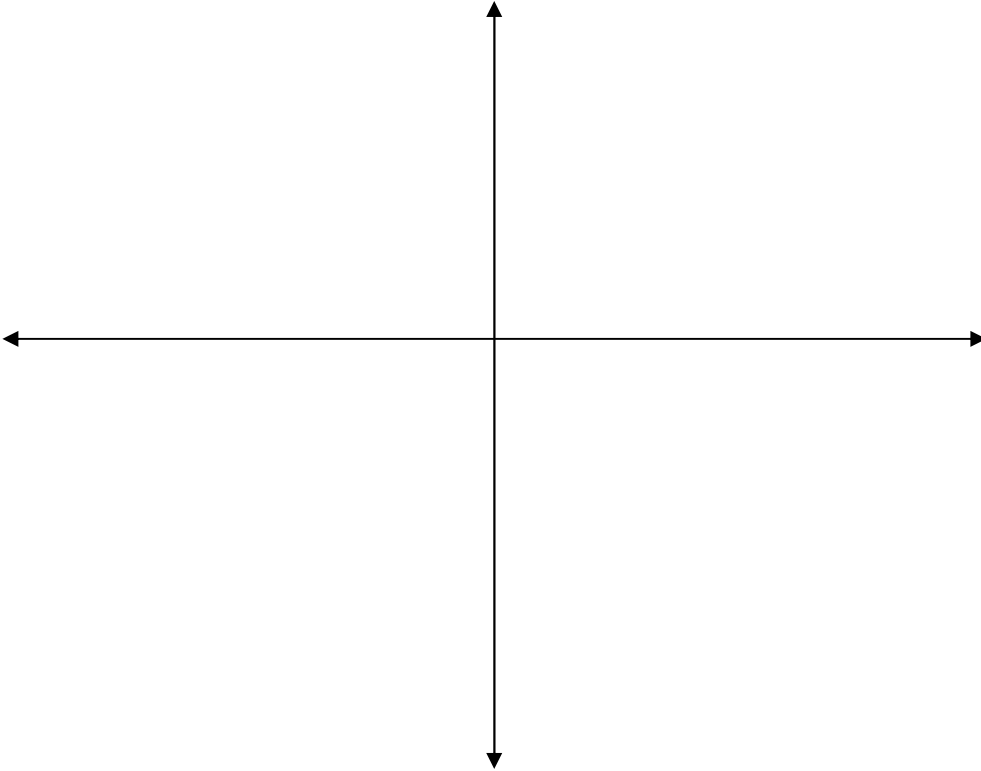
c) $\ln(2x + 3) - 2\ln(x) = 0$ x = _____

d) $9e^{3x} - 6e = e + 2e^{3x}$ x = _____

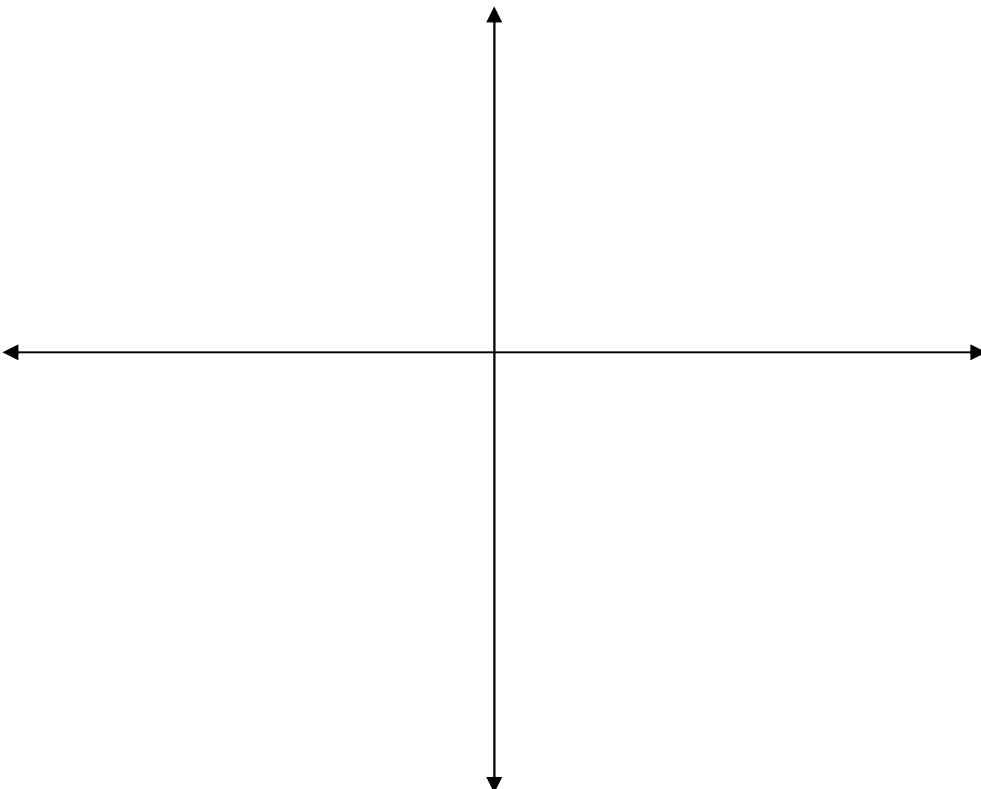
e) $5e^x = 3$ x = _____

5) (5pts each) Graph the following. Label all asymptotes and at least two points on each graph. Clearly state the transformations made from a known exponential or logarithmic function, $y = a^x$ or $y = \log_a x$.

a) $y = -3^{x+3}$



b) $y = \ln(x-1) - 1$



- 6) (5pts) In a group project in learning theory, a model for the proportion, P (calculated as a decimal), of correct responses after “ n ” trials was found to be:

$$P = \frac{0.83}{1 + e^{-0.2n}}$$

After **how many trials** will the proportion of correct responses be at least 80%?

- 7) On Mike’s day of birth, his grandparents place \$10,000 into a trust fund that yields an interest rate of 7% **compounded continuously** until the child’s eighteenth birthday.

a) (5pts) What is the balance if Mike chooses to withdraw all funds for his college education on his eighteenth birthday?

b) (5pts) How much money should Mike’s grandparents have initially deposited to produce a balance of \$40,000 on Mike’s 18th birthday?

- 8) (5pts) Use the properties of logarithms to show that the following equation is TRUE for any $x > 0$. Justify each step by citing a rule or property from Chapter 3 (i.e $\ln(u^n) = n \ln u$).

$$-3 \ln x - (\ln(x^{-4}) + \ln(xe)) = -1$$

- 9) (5pts each) Answer the following by correcting the statement or justifying your response with a short explanation, rule, property, and/or picture.

a. TRUE or FALSE: $e = \frac{271,801}{99,990}$

- b. TRUE or FALSE: You can determine the graph of $f(x) = 4^x$ by graphing $f(x) = \log_6 x$ and then reflecting it around the line $y = x$ (Identity line).

- c. Complete the statement of the identity property of logarithms:

If $\log_a x = \log_a y$, then _____.

- d. TRUE or FALSE: The sum of the logarithms of two positive numbers is equal to the logarithm of the sum of the two positive numbers.

- e. TRUE or FALSE: A Gaussian Model, $y = ae^{-\frac{(x-\mu)^2}{\sigma}}$, will never have an x-intercept.

BONUS) +5

The rational expression $\frac{-x}{x^2 + 3x + 2}$ has a Partial Fraction Decomposition, meaning it can be equivalently written as

$\frac{A}{x+2} + \frac{B}{x+1}$, where A and B are two real numbers. Follow the steps in section 2.7 of the text to find the numbers A and B.