Math 1314, EXAM #4 / Sample Final Name ________________________________________
EPCC Fall 2006

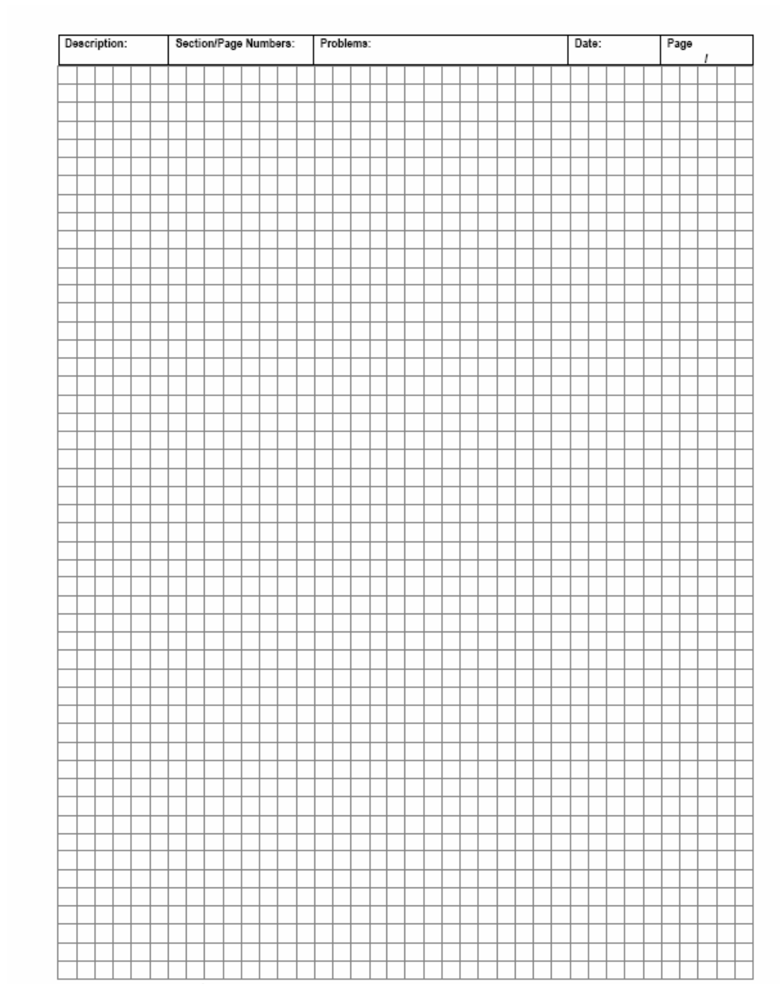
**<u>DUE Thursday, November 30, 2006</u>**

**<u>Directions</u>**:

- Please read all questions carefully.
- Answer all parts of each question.
- Please circle or box your final answers.
- Partial credit is always given for correct methods, partial correct calculations, and correct justification (rules, theorems, definitions, etc).
- NO CREDIT will be given unless there is adequate work shown or a short explanation is provided.
- Point values for each question are indicated in parentheses.

1) a. (2pts) Find the **<u>equation</u>** of the line that goes through the points $(2,4)$ and $(4,-4)$.

b. (4pts) Find the equation of the line that goes through $(-1,1)$ and is **<u>perpendicular</u>** to your answer to part a. **<u>Graph both lines below.</u>**




b. (2pts) Find the **<u>zeros</u>** of your answer to question 1a).

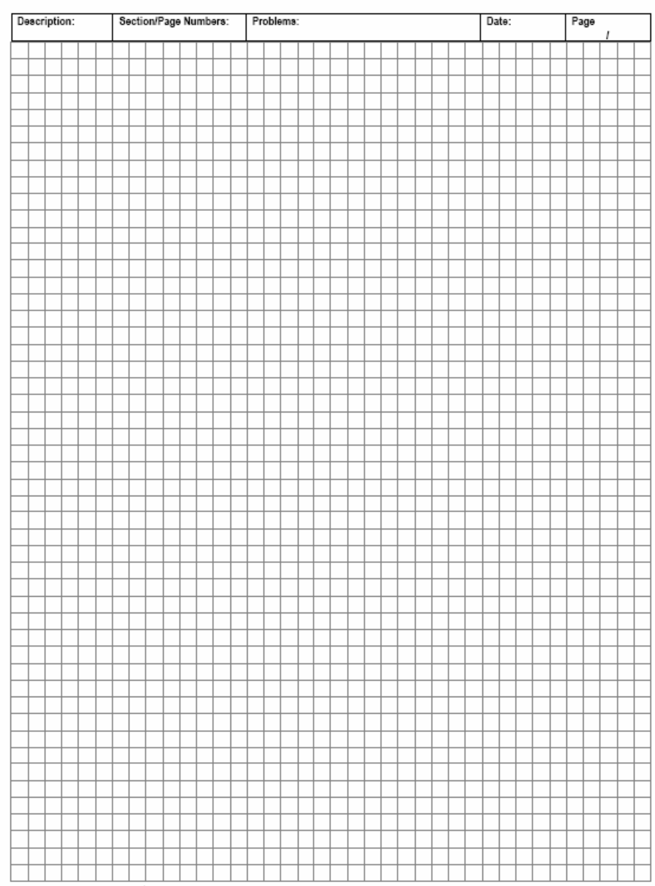
2) (8pts) Given $f(x) = x - 1$ and $g(x) = \ln(x)$, answer the following questions.

a. Find the composition function $(f \circ g)(x)$.

b. Find its inverse function, $(f \circ g)^{-1}(x)$.

c. State the **<u>domain and range</u>** of the function $(f \circ g)^{-1}(x)$ using <u>*interval notation*</u> (ex. $(-\infty, 1], [-10, 10], \ldots$)

d. Sketch the graph of $(f \circ g)^{-1}(x)$. Label any asymptotes and at least two points on the graph.



3) (6pts) Write the **Linear Factorization** of $p(x)$. (Hint: It is likely easiest to find the zeros of $p(x)$ first.)

$$p(x) = 3x^4 + 2x^3 - 7x^2 - 4x + 2$$

4) (4pts) Label the following functions as **odd**, **even**, or **neither**, and briefly **explain why** you chose your answer.

(Hint: It may be helpful to use your knowledge of graph transformations to sketch a graph of these “common functions” first.)

a. $f(x) = -|x| + 2$ ____________

b. $g(x) = x^3 - 5$ ____________

5) (6pts) Give the mathematical definition of the term **FUNCTION** by completing the definition.

A **function**, f, is a relation from an input set called the ____________________ to an output set called the ____________________ such that every element in the input set is matched with _________________ ___________ element in the output set.

6) (8pts) Solve the following for x, by finding **ALL** solutions. Do **NOT** approximate your answer.

a. $(\ln e^{-2}) + (\ln e)^{-2} + \log_{\pi} 1 + 2e^0 - 2\log_{25} 5 = x$ x = _________

b. $\log_8(x^2 + 2x) - 3\log_8(x) = 0$ x = _________

7) (6pts) Assume that the decay of radioactive "Cesium-137" obeys the Exponential Decay Model, $P(t) = P_0 e^{kt}$, where $P(t)$ is the amount of material remaining after $t$ years, $P_0$ is the initial amount of material, $k$ is the decay constant, and $t$ is time measured in years.

The half-life of "Cesium-137" is 30 years. Recall half-life is the amount of time it takes for any amount of radioactive material to decay in half.

**What percent of a present amount of "Cesium-137" will remain after 100 years?**

(Hint: You must first find the decay constant for Cesium-137. The initial amount of material is irrelevant.)

8) (6pts) Solve the following system of equations using any method (substitution, elimination, Gaussian elimination, Gauss-Jordan with matrices, etc). If the system is inconsistent or has infinitely many solutions, state so.

$$\begin{aligned} 2x + 3y &= 1 \\ -3x - 2y &= 1 \end{aligned}$$

9) The height y, in feet, of a ball thrown by a child is represented by the **Quadratic Equation** $y = -\frac{1}{10}x^2 + 2x + 3$, where x represents the horizontal distance of the ball away from the child.

a. (2pts) How high is the ball when it leaves the child's hand, i.e. when it is 0 feet away from the child?

b. (2pts) What is the **maximum height** of the ball?

c. (2pts) **Explain how** you would calculate the horizontal distance the ball travels away from the child until it hits the ground? (You do not need to calculate the value.)

10) (42 / each question = 2pts) Fill in the blanks or answer with an appropriate word, phrase, or mathematical expression.

a. List the **possible outcomes of a linear system** of equations in two variables, based on the number of possible solutions obtained. (Hint: There are three possible outcomes.)

b. Give an example of a **Rational Function** that has a **Slant Asymptote.**

c. Write a matrix of order *4x2*.

d. Explain the relationship between a **one-to-one** function and the **Horizontal Line Test**.

e. Write a **linear inequality** that contains the point $(0,0)$ in its solution region. Briefly explain how you know your answer is correct.

f. If the inverse of the $n_x n$ matrix $A$ is the matrix $B$, then using matrix multiplication $AB =$ ________ .

g. The ___________________ of a function f(x) is the set of x-values for which the function is defined.

h. A square matrix consisting of ones on its main diagonal and zeros everywhere else is called the ________________________.

i. If a system of linear equations in two variables has no solution, then the graphs of the two equations in the system will be __________________.

j. (3) Consider the chemical equation $PV = nRT$ , which relates the volume of a gas, V, with the pressure, P, and temperature, T, applied to the gas. Does the *volume* of a gas **vary directly** or **vary inversely** with the *pressure* applied to the gas?

k. Based on the average age of students in this class, you will likely retire in 40 years. Suppose you inherit $20,000 today to invest in a stock index fund averaging 10% interest compounded continuously.
**Will you become a millionaire**? Justify your answer with a calculation.

l. **True or False, justify your answer with a short explanation**:

A polynomial can have only (2+3i) as a zero.

m. **True or False, justify your answer with a short explanation**: You can determine the graph of $f(x) = 5^x$ by graphing $h(x) = \log_5 x$ and then reflecting it around the line y = x (Identity line).

n. Complete the statement of the inverse property of logarithms:

$$\log_a\left(a^x\right) = \_\_\_\_\_\_\_\_\_\_$$

o. **True or False, justify your answer with a short explanation**:

A Logistic Model, $y = \dfrac{a}{1+be^{-cx}}$ , will increase without bounds as $x \to \infty$ ("as x approached infinity").

p. If (x+1) is a factor of the polynomial f(x), then f(-1) = ______ and the point _____________ is an x-intercept of the graph of f(x).

q. **TRUE or FALSE:**

The system
$$\begin{array}{rcrcrcr} x & + & 3y & - & 6z & = & -16 \\ & & & & z & = & 1 \\ & & y & + & 3z & = & -1 \end{array}$$
is written in **Row-Echelon Form.**

If you answer True, state why.
If you answer False, correct the system so that it is in Row-Echelon Form.

r. If a polynomial f(x) is divided by (x – k), then by the **Remainder Theorem**, the remainder "r" is equal to

___________________.

s. The Gauss-Jordan method for solving a system of linear equations using matrices seeks to reduce the

coefficient matrix to ________________________________ form in order to simply "read-off" the solution.

t. What is the **y-intercept** of the graph of $y = (2.5)^x$ ?

u. The function which takes the square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ as input and returns the value $(ad - bc)$ is

called the ______________________ of matrix $A$.

BONUS +5)

Suppose your grade for the semester is based on Lab Average, Quiz Average, and Exam Average.
Both the Lab and Quizzes are weighted at 25% of your overall grade and the exams are worth 50%.
Let ‘x’ be the lab average (out of 100).
Let ‘y’ be the quiz average (out of 100).
Let ‘z’ be the exam average (out of 100).

1) The sum of all three averages equals 250.
2) One-half the lab average plus one-half the exam average add up to 85.
3) Your average for the semester is 85.

**<u>Write and solve a system of linear equations</u>** in x, y, and z to find your averages for Labs, Quizzes, and Exams.

Hint: **<u>If</u>** each grade were weighted equally (instead of 25%/25%/50%), the third statement above would correspond to the equation $\frac{1}{3}x + \frac{1}{3}y + \frac{1}{3}z = 85$.

**Quadratic Formula**: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Leading Coefficient Test**:

Let “n” be the degree and let “$a_n$” be the leading coefficient.

1. If n is even and $a_n$ is positive, the graph increases to the right and left
2. If n is even and $a_n$ is negative, the graph decreases to right and left
3. If n is odd and $a_n$ is positive, the graph increases to right and decreases to left
4. If n is odd and $a_n$ is negative, the graph decreases to right and increases to left

**Rational Zero Test**:
Let f(x) be a polynomial with integer coefficients and a non-zero constant term. Then, any rational zero of f(x) will be of the form $\dfrac{p}{q}$ where p and q have no common factors other than 1, and p is a factor of the constant term $a_o$ and q is a factor of the leading coefficient $a_n$.

**Rule of Signs**:
The polynomial f(x) must have a non-zero constant term and real coefficients.

1. The number of positive real zeros is either equal to the number of changes of sign of f(x) or less than that number by an even integer.
2. The number of negative real zeros is either equal to the number of changes of sign of f(-x) or less than that number by an even integer.

**Upper and Lower Bound Rules**
Let f(x) be a polynomial with real coefficients and a positive leading coefficient. Suppose f(x) is divided by (x – c) using synthetic division.

1. If c > 0 and each number in the last row (quotient) is either positive or zero, then c is an upper bound for the real zeros of f(x).

2. If c < 0 and the numbers in the last row (quotient) are alternately positive and negative (zero can count as either positive or negative), then c is a lower bound for the real zeros of f(x).

**Asymptotes of Rational Functions**:
Let $f(x) = \dfrac{N(x)}{D(x)}$, n be the degree of N(x) and m the degree of D(x).

1. f has Vertical Asymptotes at the zeros of D(x).

2. a. If n < m, y = 0 is a horizontal asymptote.

   b. If n = m, then $y = \dfrac{a_n}{b_m}$ is a horizontal asymptote, where $a_n$ and $b_m$ are the leading coefficients of N(x) and D(x) respectively.

   c. If n > m , then there is no horizontal asymptote.

3. If n = m+1, then there is a slant asymptote that you find using long division.